This line would like your name ___Key___

AP Calculus: Ch. 3 Review

1. A man is traveling to San Antonio. A table of values of the function v(t) are provided below for his velocity as he drives there. Answer the questions that follow using the table. <u>Use correct units.</u>

Time (s)	0	3	5	6	8
Speed (m/s)	80	72	67	89	83

a) What is the average acceleration of the car from time t=3 to t=5.

$$\frac{v(5)-v(3)}{5-3} = \frac{67-72}{2} = \boxed{\frac{-5}{2}\ m/s^2}$$

b) Estimate the instantaneous acceleration at t=2.

$$\frac{v(3)-v(0)}{3-0} = \frac{72-80}{3} = \boxed{\frac{-8}{3}\ m/s^2}$$

c) Estimate v'(6).

$$\frac{v(8)-v(6)}{8-6} = \frac{83-89}{2} = \frac{-6}{2} = \boxed{-3\ m/s^2}$$

2. Find the values of the limits below.

a) $\lim_{x\to-\infty} \frac{e^x}{5x+3} = \boxed{0}$

BoBo

b) $\lim_{x\to\infty} \frac{x+9x^2+8}{3x} = \frac{9(\infty)^2}{3(\infty)} = \frac{+}{+} \to \boxed{\infty}$

BoTi

c) $\lim_{x\to\frac{\pi}{4}} \frac{\cos(7x)}{x} = \frac{\cos\left(\frac{7\pi}{4}\right)}{\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{4\sqrt{2}}{2\pi}$

$\boxed{\frac{2\sqrt{2}}{\pi}}$

d) $\lim_{x\to1} \frac{x^2+4x-5}{x-1} = \frac{(x+5)\cancel{(x-1)}}{\cancel{(x-1)}} = x+5$

$1+5 = \boxed{6}$

3. Is the piecewise function below continuous. Show the rules as of to why it is or is not.

$x(3x+2)$

$$f(x) = \begin{cases} \frac{3x^2+2x}{\cancel{x}}, & x \le 0 \\ e^x, & x > 0 \end{cases}$$

1 $\lim_{x\to0}$ $3(0)+2 = \boxed{2}$ not the same

$e^0 = \boxed{1}$

$\lim_{x\to0}$ DNE

not continuous!

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4. Using the function $f(x) = \tan(4x)$. Find the instantaneous rate of change of $f(x)$ at $x = \frac{\pi}{3}$

$$f'(x) = \sec^2(4x) \cdot 4$$

$$f'(\tfrac{\pi}{3}) = \sec^2(\tfrac{4\pi}{3}) \cdot 4 = \left(\frac{-2}{\sqrt{1}}\right)^2 \cdot 4 = \boxed{16}$$

5.

x	-2	-1	0	1	2
f(x)	1	-2	-6	5	0
h(x)	-1	-5	0	2	9
f '(x)	2	9	-1	-2	0

1. Let $f(x)$ be a differentiable function and $g(x) = f^{-1}(x)$. Using the table above answer the questions below.
   a. Find $g'(1)$

$$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(-2)} = \boxed{\frac{1}{2}}$$

$g(1) = -2$ $f(-2) = 1$
$g(-2) = -1$ $f(-1) = -2$
$f(0) = -6$
$f(1) = 5$
$f(2) = 0$

   b. Find $g'(-2)$

$$g'(-2) = \frac{1}{f'(g(-2))} = \frac{1}{f'(-1)} = \boxed{\frac{1}{9}}$$

g

6. Find $\frac{dy}{dx}$ of $x^2 + 4xy^2 = 12x$ at $(-2, 1)$

$$2x + (y^2 4 + 4x 2y \tfrac{dy}{dx}) = 12$$

$$2x + 4y^2 + 8xy\tfrac{dy}{dx} = 12$$

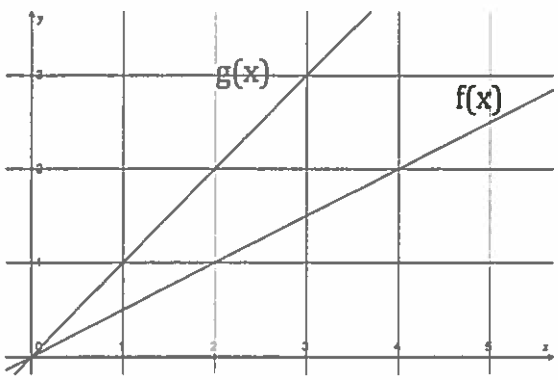
$$8xy\tfrac{dy}{dx} = 12 - 2x - 4y^2$$

$$\boxed{\frac{dy}{dx} = \frac{12 - 2x - 4y^2}{8xy}}$$

$$\frac{12 - 2(-2) - 4(1)^2}{8(-2)(1)}$$

$$\frac{12 + \cancel{4} - \cancel{4}}{-16} = \frac{12}{-16} = \frac{-6}{8} = \boxed{\frac{-3}{4}}$$

7. Using the graph, find h'(2).



a) $h(x) = g(x)^3$

slope of g at x=2

$$h'(x) = 3g(x)^2 \cdot g'(x)$$

$$h'(2) = 3g(2)^2 g'(2) = 3(2)^2 \cdot (1) = \boxed{12}$$

b) $h(x) = \frac{\sqrt{g(x)}}{f(x)} = \frac{g(x)^{1/2}}{f(x)}$

$$h'(x) = \frac{f(x)\frac{1}{2}g(x)^{-1/2} \cdot g'(x) - g(x)^{1/2} f'(x)}{f(x)^2}$$

$$h'(2) = \frac{\frac{(1)(\frac{1}{2})(1)}{\sqrt{2}} - \sqrt{2}(\frac{1}{2})}{(1)^2} = \frac{1}{2\sqrt{2}} - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{1-2}{2\sqrt{2}}$$

$$\boxed{\frac{-1}{2\sqrt{2}}}$$