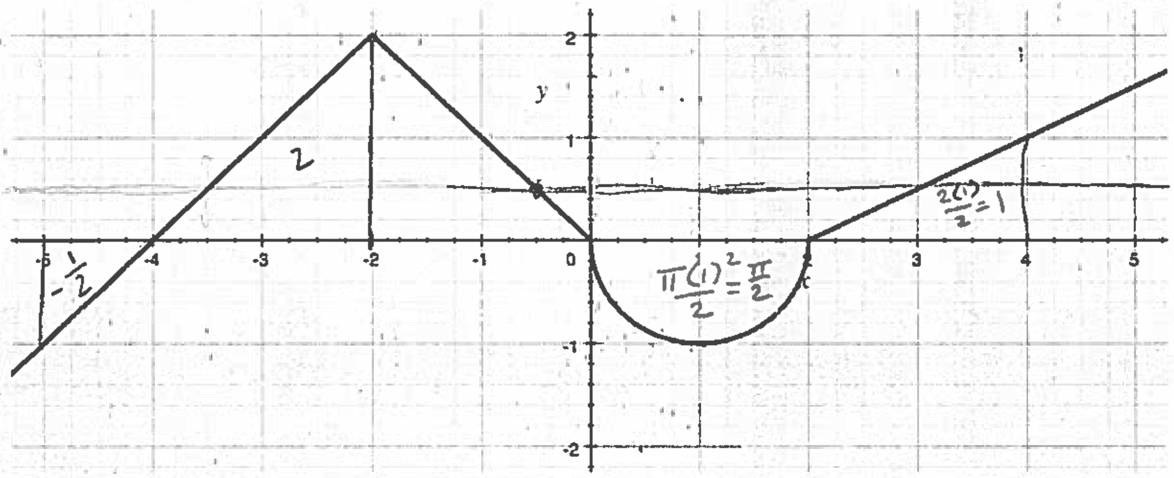
Review your name after writing it: Key

AP Calculus AB: Ch.8 Review



1. The graph above is a graph of $f(x)$. The function $g(x)$ is given by the formula $g(x) = -\frac{x}{2} + \int_{-5}^{x} f(x)\,dx$.
   a) Evaluate $\int_0^4 f(x)\,dx$ and $\int_0^4 f(x) + 2\,dx$
   b) Find $g'(x)$.
   c) Evaluate $g(-2)$, $g'(-2)$, and $g''(-2)$
   d) Find when the graph of $g(x)$ has a critical value and label each as either maximum or minimum values. Justify.
   e) Find when the graph of $g(x)$ has a point of inflection. Justify.

a) $\int_0^4 f(x)\,dx = \boxed{-\frac{\pi}{2}+1}$

$\left(\int_0^4 f(x)\,dx\right) + \int_0^4 2\,dx = \left(-\frac{\pi}{2}+1\right) + \left[2x\right]_0^4 = -\frac{\pi}{2}+1+2(4) = \boxed{-\frac{\pi}{2}+9}$

b) $\boxed{g'(x) = -\frac{1}{2} + f(x)}$

c) $g(-2) = -\frac{(-2)}{2} + \int_{-5}^{-2} f(x) = +1 + -\frac{1}{2} + 2 = \frac{+2}{2} - \frac{1}{2} + \frac{4}{2} = \boxed{\frac{5}{2}}$

$g'(-2) = -\frac{1}{2} + f(-2) = -\frac{1}{2} + 2 = -\frac{1}{2} + \frac{4}{2} = \boxed{\frac{3}{2}}$

$g''(-2) = f'(-2) = \boxed{\text{und}}$

~~2. If $g(x) = \int_{2t}^{t^3} \ln(2x)\,dx$ then find $g'(t)$ and $g'(1)$.~~

d) $g'(x) = 0$

$-\frac{1}{2} + f(x) = 0$

$f(x) = \frac{1}{2}$

$x = -3.5, -.5, 3$

max at $x = -.5$ because graph of $f(x)$ switched from $f(x) > .5$ to $f(x) < .5$

min at $x = 3, -3.5$ because $f(x)$ switched from $f(x) < .5$ to $f(x) > .5$

e) $g''(x) = 0$

$f'(x) = 0$

$x = -2, 1$

this is where the graph of $f(x)$ slope changed sign.

Review your name after writing it:____

Calculator allowed on this problem

2. A particle's velocity is given by the function $v(t) = e^{2t}\cos\left(\frac{t}{2}\right)$ for the interval $0 \leq t \leq 5$ and it's initial position at $x(0) = -2$
   a) Find the position of the particle at $t = 3$
   b) Find the distance traveled for the interval $0 \leq t \leq 5$
   c) Find the average velocity of the particle over the interval $0 \leq t \leq 5$
   d) Find the average acceleration of the particle for the interval $0 \leq t \leq 5$
   e) Find the instantaneous acceleration of the particle at $t = 3$
   f) Is the particle speeding up or slowing down at $t = 3$

a) $-2+\int_0^3 v(t)\,dt = 58.3$

b) $\int_0^5 |v(t)|\,dt = 6878.85$

c) $\frac{\int_0^5 v(t)\,dt}{5-0} = -1350.76$

d) $\frac{v(5)-v(0)}{5-0} = -3529.47$

e) $v'(3) = -144.134$

f) $v(3) = 28.537$
$a(3) = -144.134$
slowing down $v(3)$ and $a(3)$ are different signs.

Evaluate each of the following integrals below

3. $\int_0^2 xe^{x^2}\,dx$

$u = x^2$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\int_0^4 \cancel{x} e^u \frac{du}{2\cancel{x}}$

$\int_0^4 \frac{e^u}{2}\,du = \left[\frac{e^u}{2}\right]_0^4 = \frac{e^4}{2} - \frac{e^0}{2} = \boxed{\frac{e^4}{2} - \frac{1}{2}}$

4. $\int_{-1}^0 9x(3x^2-2)^2$

$u = 3x^2 - 2$
$du = 6x\,dx$
$dx = \frac{du}{6x}$

$\int_1^{-2} 9\cancel{x}(u)^2 \frac{du}{6\cancel{x}}$

$-\int_{-2}^1 \frac{3}{2}u^2\,du = -\left[\frac{3u^3}{2(3)}\right]_{-2}^1 = -\left[\frac{u^3}{2}\right]_{-2}^1$

$= -\left[\frac{1^3}{2} - \frac{(-2)^3}{2}\right] = -\left[\frac{1}{2} + \frac{8}{2}\right] = \boxed{-\frac{9}{2}}$

5. Evaluate the integral $\int_a^{2b} f(x) - 3\,dx$ if you know $\int_a^{2b} f(x)dx = b + 3a$

$\int_a^{2b} f(x)\,dx - \int_a^{2b} 3\,dx$

$(b+3a) - [3x]_a^{2b} = b + 3a - [3(2b) - 3(a)] = b + \cancel{3a} - 6b + \cancel{3a} = \boxed{-5b}$

6. If $g(x) = \int_{2t}^{t^3} \ln(2x)\,dx$ then find $g'(t)$ and $g'(1)$.

$3t^2 \ln(2t^3) - 2\ln(2(2t))$

$\boxed{3t^2 \ln(2t^3) - 2\ln(4t)}$