

U are? Key

AP Calculus AB: All Integrals

1. $\int \frac{6x^3 - \frac{4}{3}x + 8}{2\sqrt{x}} dx$

$\int 3x^{5/2} - \frac{2}{3}x^{1/2} + 4x^{-1/2} dx$
 $2 \cdot \frac{3x^{7/2}}{7} - \frac{2 \cdot 2x^{3/2}}{3 \cdot 3} + \frac{2 \cdot 4x^{1/2}}{1} + C = \frac{6}{7}x^{7/2} - \frac{4x^{3/2}}{9} + 8\sqrt{x} + C$

2. $\int \frac{6}{\sqrt[3]{x^2}} - \frac{14}{2}\sqrt[4]{x^3} + \frac{6}{x} dx = \int 6x^{-2/3} - 7x^{3/4} + 6x^{-1} dx$

$3 \cdot \frac{6x^{1/3}}{1} - \frac{4 \cdot 7x^{7/4}}{7} + 6 \ln|x| + C$
 $18\sqrt[3]{x} - 4x^{7/4} + 6 \ln|x| + C$

3. $\int \frac{9x^2 - 3}{x^3 - x + 5} dx$ $u = x^3 - x + 5$
 $du = 3x^2 - 1 dx$

$\int \frac{3(3x^2 - 1) du}{u \cdot 3x^2 - 1}$ $dx = \frac{du}{3x^2 - 1}$
 $\int \frac{3}{u} du = 3 \ln|u| + C = 3 \ln|x^3 - x + 5| + C$

4. $\int \frac{(\sqrt{x}-4)^3}{\sqrt{x}} dx$ $u = \sqrt{x} - 4 = x^{1/2} - 4$
 $du = \frac{1}{2}x^{-1/2} dx = \frac{1}{2\sqrt{x}} dx$

$\int \frac{u^3}{\sqrt{x}} du \cdot 2\sqrt{x}$ $dx = du \cdot 2\sqrt{x}$
 $\frac{2u^4}{4} + C = \frac{u^4}{2} + C = \frac{(\sqrt{x}-4)^4}{2} + C$

5. $\int 12x^2 \sqrt{2x^3 - 8} dx$ $u = 2x^3 - 8$
 $du = 6x^2 dx$

$\int 12x^2 (u)^{1/2} \frac{du}{6x^2}$ $dx = \frac{du}{6x^2}$
 $2 \cdot \frac{2u^{3/2}}{3} + C = \frac{4}{3}x^{3/2} + C$

6. $\int \frac{4x}{3(x^2-7)^3} dx$ $u = x^2 - 7$
 $du = 2x dx$

$\int \frac{4x}{3u^3} \frac{du}{2x}$ $dx = \frac{du}{2x}$
 $\frac{4}{6} \int u^{-3} du = \frac{2}{3} \frac{u^{-2}}{-2} + C = \frac{-1}{3u^2} + C$

7. $\int \frac{\sec^2(x)}{10 - \tan(x)} dx$ $u = 10 - \tan(x)$
 $du = -\sec^2(x) dx$

$\int \frac{\sec^2(x) du}{u \cdot -\sec^2(x)}$ $dx = \frac{du}{-\sec^2(x)}$
 $-\ln|u| + C = -\ln|10 - \tan(x)| + C$

8. $\int \frac{\sin(x)}{\sqrt{\cos(x)}} dx$ $u = \cos(x)$
 $du = -\sin(x) dx$

$\int \frac{\sin(x) du}{u^{1/2} \cdot -\sin(x)}$ $dx = \frac{du}{-\sin(x)}$
 $-\int u^{-1/2} du = -\frac{2u^{1/2}}{1} = -2u^{1/2} + C = -2\sqrt{\cos(x)} + C$

9. $\int x(x-5)^3 dx$ $u = x-5$
 $du = dx$

$\int (u+5)u^3 du$ $dx = du$
 $\int u^4 + 5u^3 du$ $x = u+5$
 $\frac{u^5}{5} + \frac{5u^4}{4} + C = \frac{(x-5)^5}{5} + \frac{5(x-5)^4}{4} + C$

10. $\int 12x \sec(4x^2) \tan(4x^2) dx$ $u = 4x^2$
 $du = 8x dx$

$\int 12x \sec(u) \tan(u) \frac{du}{8x}$ $dx = \frac{du}{8x}$
 $\frac{3}{2} \int \sec(u) \tan(u) du = \frac{3}{2} \sec(4x^2) + C$

11. $\int \frac{4 \tan(x)}{\cos(x)} - 5 dx$

$\int 4 \tan(x) \sec(x) - 5 dx$
 $4 \sec(x) - 5x + C$

12. $\int \frac{e^x}{6x^2}$ $u = \frac{x}{3} = 3x^{-1}$
 $du = -3x^{-2} = -\frac{3}{x^2} dx$

$\int \frac{e^u}{6x^2} \cdot \frac{du}{-3}$ $dx = \frac{x^2 du}{-3}$
 $\int \frac{e^u}{-18} du = -\frac{e^u}{18} + C = -\frac{e^{3/x}}{18} + C$