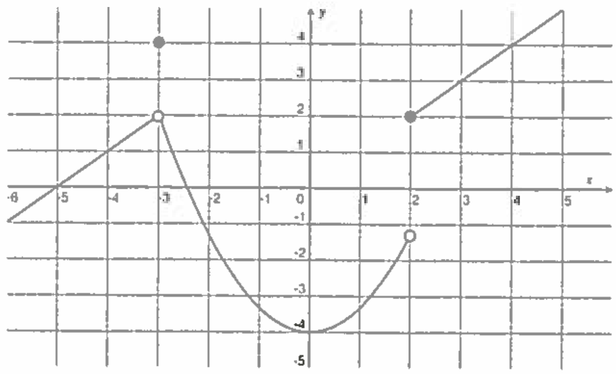
You're done with half of Calculus!! Who's excited!?!?! Key

AP Calculus AB: Ch. 5 Test



1. Using the graph above, circle the statements below that are true.

a) $\lim_{x \to -3} f(x) = 3$

b) $\lim_{x \to -3} f(x) = f(2)$

c) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

d) The function is continuous at $x = 0$

e) $\lim_{x \to -4} f'(x) = \lim_{x \to 3} f'(x)$

f) The function is continuous at $x = -3$

2. $\lim_{x \to -\infty} \frac{e^x + 3}{x^2}$ $= \boxed{0}$

3. $\lim_{x \to a} \frac{x-a}{x^3 - a^3}$

$\frac{1}{3x^2}$ $\boxed{\frac{1}{3a^2}}$

4. $\lim_{x \to 1} \frac{\sin(1-x)}{\ln(x)}$

$\frac{x\cos(1-x)\cdot(-1)}{1}$ $= \boxed{-1}$

5. $\lim_{x \to \infty} 2^{1/x}$ $= \boxed{1}$

6. Find the k-value to make the function, $g(x) = \begin{cases} kx^2 - 2x, & x > 2 \\ 3x + 1, & x \le 2 \end{cases}$ continuous.

$k(2)^2 - 2(2) = 3(2) + 1$

$4k - 4 = 7$

$4k = 11$

$\boxed{k = \frac{11}{4}}$

7. Calculate the x-value when $f(x) = \arctan(2x)$ has a slope of 2/3.

$\frac{2}{1+4x^2} = \frac{2}{3}$

$\boxed{x = \sqrt{\frac{1}{2}}}$

$6 = 2 + 8x^2$

$4 = 8x^2$

$\frac{1}{2} = x^2$

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8. A car's velocity is given in the table below over a 7 second interval, answer the questions that follow. The car's velocity is both continuous and differentiable.

Time(s)	0	1	3	4	7
velocity(m/s)	80	72	67	89	90

a. Calculate the average acceleration on the interval [1,4]

$$\frac{89-72}{4-1} = \boxed{\frac{17}{3}\ m/s^2}$$

b. Does the car ever travel a velocity of 70 m/s? Justify.

yes b/c of IVT

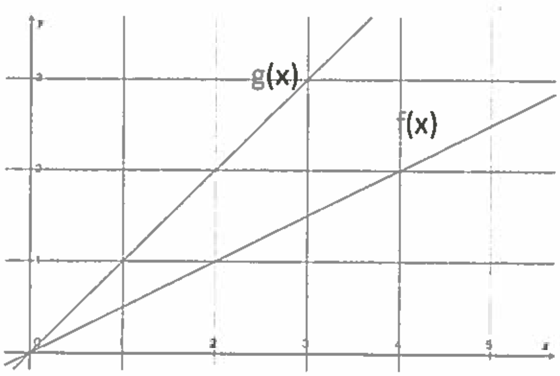
c. Over the interval [0,1] does the car ever have an acceleration of -8 m/s^2? Justify.

$\frac{72-80}{1-0} = -8\ m/s^2$ yes by MVT

d. How many times must the acceleration of the car be zero? Justify.

at least once between [3,4]
b/c velocity changed from dec to inc.

9. Using the graph, find h'(2). If the function h(x) is given by $h(x) = \sqrt{f(x)^3}$



$$h'(x) = \frac{3\sqrt{f(x)}\cdot f'(x)}{2}$$

$$h'(2) = \frac{3\sqrt{1}\cdot\frac{1}{2}}{2} = \boxed{\frac{3}{4}}$$

10. Let $f(x)$ be a differentiable function with an inverse function $g(x)$ and have the values $f(0) = 1, f(1) = 3, f'(3) = -3, and\ f'(1) = 4$. Find $g'(3)$.

$$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \boxed{\frac{1}{4}}$$

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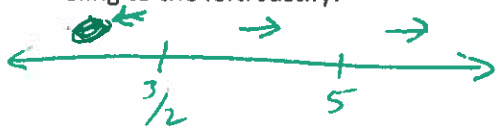
11. The function $v(t)$ describes the velocity of a bug as it travels through the air for a time interval of [0,10] seconds. Using the function $v(t) = (t-5)^2(2t-3)$ answer the questions below if velocity is measured in m/s.

a) Calculate the interval where the bug is traveling to the left. Justify.

$v(t)=0$

$(t-5)^2=0 \quad 2t-3=0$

$t=5 \quad t=\frac{3}{2}$



$v(0)=(+)(-)=+$

$v(2)=(+)(+)=+$

$v(10)=(+)(+)=+$

from $[0, 3/2]$ seconds because $v(t)$ is negative

b) Identify where the bug changed direction. Justify.

@ $t=3/2$ because the $v(t)$ changed signs.

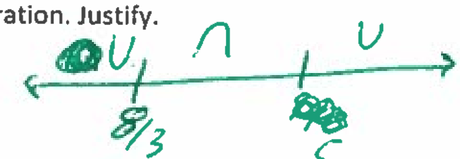
c) Calculate the interval where the bug has negative acceleration. Justify.

$a(t)=(t-5)^2(2)+(2t-3)2(t-5)$

$=(t-5)(2)[t-5+2t-3]$

$=2(t-5)(3t-8)$

$a(t)=0 \quad t=5 \quad t=8/3$



$a(0)=(-)(-)=+$

$a(4)=-+=-$

$a(6)=++=+$

$(0, 8/3)$ & $(5,10)$ because $a(t)$ is negative.

d) Is the bug speeding up or slowing down at t=4? Justify.

$v(4)=(+)(+)=+$

$a(4)= -$

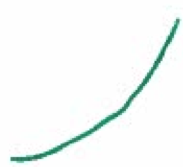
slowing down since $v(4)$ and $a(4)$ are different signs.

e) Calculate the bugs average acceleration on the interval [3,5].

$$\frac{v(5)-v(3)}{5-3} = \frac{0-4(3)}{2} = \boxed{-6 \ m/s^2}$$

12.If the function $f(x)$ is twice-differentiable and $f'(x) > 0$ and $f''(x) > 0$ for all $x > 1$ and the value $f(1) = 2$ and $f(2) = 3$ are points on the function $f(x)$. Circle all the values below that are possible.

I. $f(3) = 1$
II. $f(3) = 4$
III. $f(3) = 6$
IV. $f(3) = -2$
V. $f(3) = 8$



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calculator

13. A person is piling rocks up while a machine takes them away. A function of the number of pounds of rocks in the pile with respect to time, in hours, is given by the function $P(t) = 4t(3t-2)^3$

a) what is the average rate of change of the lbs of rocks in the pile for the interval [0,1]

$$\frac{P(1)-P(0)}{1-0} = 4\ \frac{lbs}{hr}$$

b) What is the instantaneous rate of change at t=1.

$$P'(1) = 40\ \frac{lbs}{hr}$$

c) What is the absolute ~~minimum~~ max value for the number of pounds of rocks for the interval [0,1].

0	0
1	4
2/3	0

$$P'(x) = \frac{2}{3}$$

$P(0) = 0$

$P(1) = 4$

$P(2/3) = 0$

15. Find the $\frac{dy}{dx}$ for the function $4xy^2 = y + 3x^2$

$$4x2y\frac{dy}{dx} + y^2 4 = \frac{dy}{dx} + 6x$$

$$8xy\frac{dy}{dx} - \frac{dy}{dx} = -4y^2 + 6x$$

$$\boxed{\frac{dy}{dx} = \frac{-4y^2+6x}{8xy-1}}$$

~~Calculator allowed passed this point~~

16. Using the equation for the tangent line of the function $f(x) = \ln(.432x) + 2$ at $x = 3$, estimate when the function $f(x)$ equals 4.

$$y - 2.25 = \frac{1}{3}(x-3)$$

$$4 = \frac{1}{3}(x-3) + 2.259$$

$$\boxed{x = 8.223}$$

17. If a particles velocity is given by the function $v(t) = \sin(5t^2) - 0.5$, how many times did the particle change directions on the interval [0,2]? Justify.

7 times