Look at that long empty line: Key

AP Calculus AB: 9.7 Differential Equations

$u = y-3$
$dy = dy$
$dy = du$

1. Find the particular solution of the equation $\frac{dy}{dx} = x^3(y-3)$ with the initial condition $f(0) = 5$.

$\int \frac{1}{y-3}\,dy = \int x^3\,dx$

$\int \frac{1}{u}\,du = \frac{x^4}{4} + C$

$\ln|u| = \frac{x^4}{4} + C$

$\ln|y-3| = \frac{x^4}{4} + C$

$\ln|2| = \frac{(0)^4}{4} + C$

$C = \ln(2)$

$\ln|y-3| = \frac{x^4}{4} + \ln(2)$

$y - 3 = e^{x^4/4 + \ln(2)}$

$y - 3 = e^{x^4/4} e^{\ln(2)}$

$y - 3 = 2e^{x^4/4}$

$\boxed{y = 2e^{x^4/4} + 3}$

2. Consider the differential equation $\frac{dy}{dx} = (x^2 - 3)(3 - y)$ with the initial condition $f(3) = 1$.

   a. Write an equation for the line tangent to the function $f(x)$ at $x = 3$.

$(y - 1) = 12(x - 3)$ $\quad m = (3^2 - 3)(3 - 1) = 6(2) = 12$

$y - 1 = 12x - 36$

$\boxed{y = 12x - 35}$

   b. Use the tangent line from part (a) to linear approximate the value of $f(2.1)$.

$y = 12(2.1) - 35$

$= \boxed{-9.8}$

   c. Find the particular solution of the differential equation.

$u = 3 - y$
$du = -dy$
$dy = -du$

$\int \frac{1}{3-y}\,dy = \int x^2 - 3\,dx$

$\int -\frac{1}{u}\,du = \frac{x^3}{3} - 3x + C$

$-\ln|u| = \frac{x^3}{3} - 3x + C$

$-\ln|3-y| = \frac{x^3}{3} - 3x + C$

$-\ln|2| = \cancel{9 - 9} + C$

$C = -\ln(2)$

$-\ln|3-y| = \frac{x^3}{3} - 3x - \ln(2)$

$3 - y = e^{x^3/3 - 3x - \ln(2)}$

$-y = 2e^{x^3/3 - 3x} - 3$

$\boxed{y = -2e^{x^3/3 - 3x} + 3}$

3. Consider the differential equation $\frac{dy}{dx} = \sin(x) - y$ with the initial condition $f(0) = 1$

   a. Find the particular solution of the differential equation.

$\int y\,dy = \int \sin(x)\,dx$

$\frac{y^2}{2} = -\cos(x) + C$

$\frac{(1)^2}{2} = -\cos(0) + C$

$\frac{1}{2} = -1 + C$

$\frac{3}{2} = C$

$\frac{y^2}{2} = -\cos(x) + \frac{3}{2}$

$y^2 = 2(-\cos(x) + \frac{3}{2})$

$y^2 = -2\cos(x) + 3$

$\boxed{y = \sqrt{-2\cos(x) + 3}}$

   b. Is the function $f(x)$ concave up or concave down at $x = \frac{\pi}{2}$

$\frac{d^2y}{dx^2} = \cos(x) - y'$

$\frac{d^2y}{dx^2} = \cos(x) - (\sin(x) - y)$

$\cos(\pi/2) - \sin(\pi/2) + \sqrt{-2\cos(\pi/2) + 3}$

$\cancel{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}} + \sqrt{-\frac{2\sqrt{2}}{2} + 3} = \boxed{\sqrt{-\sqrt{2} + 3}}$

$\boxed{\text{concave up}}$