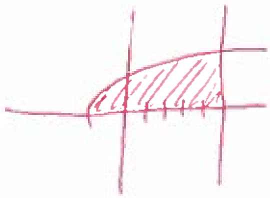
Write your name on this horizontal cross section: Key

AP Calculus AB: 9.6 Volume with Cross Sections

1. Let region S be the region enclosed by the functions $g(x) = \sqrt{x-1}$, $y = 0$, and $x = 5$. Region S is the base of a solid. For this solid, each cross section is a square perpendicular to the x-axis. Find the volume of this solid.

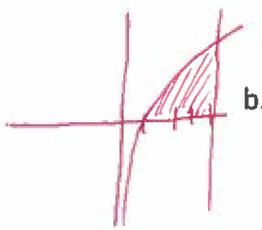


$$\int_{-1}^{5} (\sqrt{x-1})^2\,dx = \int_{-1}^{5} x-1\,dx = \left[\frac{x^2}{2}-x\right]_{-1}^{5} =$$

$$= \left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}+1\right)\right] = \left[\frac{15}{2} - -\frac{1}{2}\right] = \frac{16}{2} = \boxed{8}$$

2. (Calculator) Let R be the region enclosed by the functions $f(x) = \ln(x)$, $y = 0$ and $x = 4$.
   a. Region R is the base of a solid. For this solid, each cross section is a square perpendicular to the x-axis. Find the volume of this solid.

$$\int_{1}^{4} (\ln(x))^2\,dx = \boxed{2.597}$$



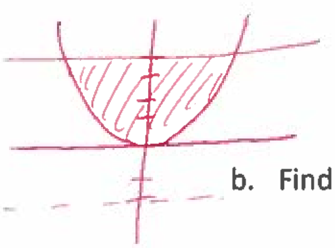
   b. Region R is the base of a solid. For this solid, each cross section is a isosceles right triangle perpendicular to the x-axis with one of the legs as the base. Find the volume of this solid.

$$\int_{1}^{4} \frac{(\ln(x))(\ln(x))}{2}\,dx = \int_{1}^{4} \frac{\ln(x)^2}{2}\,dx = \boxed{1.298}$$

   c. Region R is the base of a solid. For this solid, each cross section is semi-circle perpendicular to the y-axis. Find the volume of this solid.

$$\int_{0}^{\ln(4)} \frac{\pi\left(\frac{4-e^y}{2}\right)^2}{2}\,dy = \boxed{2.231}$$

3. (Calculator) Let region R be the region enclosed by the functions $f(x) = x^2$ and $y = 4$.
   a. Find the **area** of region R.



$$\int_{-2}^{2} 4-x^2\,dx = \boxed{10.667}$$

   b. Find the **volume** of the region R rotated about the $y = -2$ axis.

$$\pi\int_{-2}^{2} (4--2)^2-(x^2--2)^2\,dx = \boxed{294.891}$$

   c. Region R is the base of a solid. For this solid, each cross section is a semi-circle perpendicular to the x-axis with one of the legs as the base. Find the volume of this solid.

$$\int_{-2}^{2} \frac{\pi\left(\frac{4-x^2}{2}\right)^2}{2}\,dx = \boxed{6.323}$$