Put your name between these lines | Koy |

AP Calculus AB: 9.1 Area between two curves

1. Let the functions f and g be defined by $f(x) = x + 2$ and $g(x) = -x^2 + 2x + 8$.
   a. Find the intersection of the two functions algebraically.

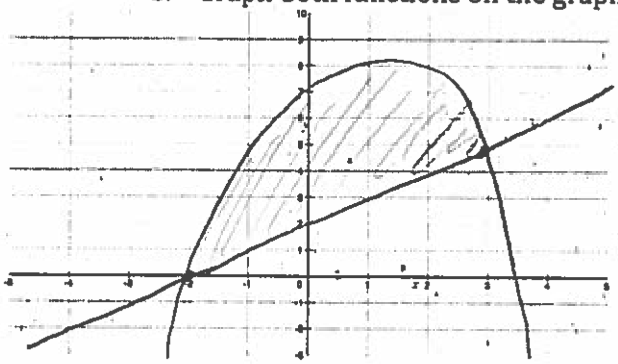
$x+2 = -x^2 + 2x + 8$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

   b. Graph both functions on the graph provided.



   c. Write an integral that will find the area enclosed by the two functions f and g and evaluate the integral.

$\int_{-2}^{3} g(x) - f(x)\,dx = \int_{-2}^{3} -x^2+2x+8 - (x+2)\,dx =$

$\int_{-2}^{3} -x^2+2x+8-x-2\,dx$

$= \int_{-2}^{3} -x^2 + x + 6\,dx$

$= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 6x\right]_{-2}^{3} = \boxed{\frac{125}{6}}$

Calculator Allowed

2. Let the functions f and g be defined by $f(x) = \frac{1}{x}$ and $g(x) = -x^2 + 2x + 5$.
   a. Find the intersections of the two functions f and g in the first quadrant.

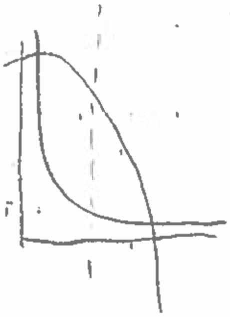
$f(x) = g(x)$

$x = .187$ & 3.388

   b. Let R be the region in the first quadrant enclosed by the functions f and g. Set up an integral to calculate the area of region R. Using your calculator, calculate the area of region R.

$\int_{.187}^{3.388} g(x) - f(x)\,dx = \boxed{11.591}$

K



   c. If the vertical line $x = k$ divides the region R into two regions of equal area, what is the value of k?

$\int_{.187}^{K} g(x) - f(x)\,dx = \int_{K}^{3.388} g(x) - f(x)\,dx$

$\boxed{k = 1.543}$

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3. Find the area bounded by the graphs $f(x) = x^2 - 2x - 4$ and $g(x) = 4x - 1$.

$$\int_{-.464}^{6.464} g(x) - f(x)\, dx = 55.426$$

4. Find the area bounded by the function $h(x) = \frac{1}{x+2}$, the x-axis and the lines x=3 to x=5.

$u = x+2$
$du = dx$

$$\int_3^5 \frac{1}{x+2}\, dx = \int_5^7 \frac{1}{u}\, du = \left[\ln(u)\right]_5^7 = \ln(7) - \ln(5)$$
$$= \ln\left(\frac{7}{5}\right)$$

5. Find the area of the region bounded by the lines x=0, x=3, the x-axis and the function $y = e^{\frac{x}{3}}$.

$u = x/3$
$du = \frac{1}{3}dx$
$dx = 3du$

$$\int_0^3 e^{x/3}\, dx = \int_0^1 e^u\, 3du = \int_0^1 3e^u\, du = \left[3e^u\right]_0^1$$
$$3e^1 - 3e^0 = \boxed{3e-3}$$
or
$$\boxed{3(e-1)}$$

Free Response (No calculator)

6. Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for the interval $1 \leq x \leq 9$

a) Find the area of R.

$$\int_1^9 \frac{1}{\sqrt{x}}\, dx = \int_1^9 x^{-1/2}\, dx = \left[\frac{2x^{1/2}}{1}\right]_1^9 = 2\sqrt{9} - 2\sqrt{1} = 6 - 2 = \boxed{4}$$

b) If the line x=k divides the region R into two regions of equal area, what is the value of k?

$$\int_1^K \frac{1}{\sqrt{x}}\, dx = \int_K^9 \frac{1}{\sqrt{x}}\, dx$$

$$\left[2\sqrt{x}\right]_1^K = \left[2\sqrt{x}\right]_K^9 \rightarrow 2\sqrt{K} - 2\sqrt{1} = 2\sqrt{9} - 2\sqrt{K}$$
$$4\sqrt{K} = 8$$
$$\boxed{K = 4}$$