Who's there?__________________________________

AP Calculus AB: Real-World FTC

Set-up the integral and use a calculator to find the answer to questions #1&2

1. If $f'(x) = 3 + \frac{1}{x^2}$ and $f(1) = 2$, find $f(2)$. *(answer: $\frac{11}{2}$)*

$$2 + \int_1^2 3 + \frac{1}{x^2}\,dx = \frac{11}{2}$$

2. If $y' = \frac{2}{x+3}$ and $f(0) = 5$, find $f(3)$. *(answer:5+ln|4|)*

$$5 + \int_0^3 \frac{2}{x+3}\,dx = 5 + 2\ln|2| = 5 + \ln|4|$$

You need a calculator for the rest of the worksheet

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed form the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

$\int_0^{12} H(t)\,dt = \boxed{70.571 \text{ gallons}}$

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

$H(6) = 5.395 \frac{gal}{hr}$

$R(6) = 8.319 \frac{gal}{hr}$

oil is falling because it is being pumped out faster than in.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

$125 + \int_0^{12} H(t)\,dt - \int_0^{12} R(t)\,dt = \boxed{122.026 \text{ gallons}}$

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

$H(t) - R(t) = 0$

$t = 4.79$ & 11.3

$+ \quad - \quad +$

$4.79 \qquad \boxed{11.3}$

min

or

Graph $H(t) - R(t)$ and check where it goes from negative to positive values.

$\boxed{t = 11.3}$

Who's there?________________________________

2005 AP® CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 4$.

   (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

   (c) Find the position of the particle at time $t = 2$.

   (d) Find the average speed of the particle over the interval $0 \le t \le 2$.

a) $v'(t) = a(t) = \dfrac{2t-3}{t^2-3t+3}$

$a(4) = \dfrac{2(4)-3}{(4)^2-3(4)+3} = \boxed{\dfrac{5}{7}}$

c) $8 + \int_0^2 v(t)\,dt = \boxed{8.369}$

d) $\dfrac{\int_0^2 v(t)\,dt}{2} = \boxed{.184}$

b) $v(t) = 0$

$\ln(t^2-3t+3) = 0$

$t^2-3t+3 = e^0$

$t^2-3t+3 = 1$

$t^2-3t+2 = 0$

$(t-2)(t-1) = 0$

$t=2 \quad t=1$

$+ \quad - \quad +$

$1 \quad 2$

$v(0) = +$

$v(1.5) = -$

$v(3) = +$

Changes direction at $t=1$ & $t=2$

traveling to the left from $[1,2]$

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\dfrac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

Calc

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

a) $G'(5) = \boxed{-24.588 \ \frac{\text{tons}}{\text{hr}^2}}$ the acceleration of how fast gravel is arriving at $t=5$

b) $\int_0^8 G(t)\,dt = \boxed{825.551 \text{ tons}}$

c) $G(5) = \boxed{98.1408 \text{ tons/hr}}$ and it's being processed at 100 tons/hr so the amount of gravel is decreasing at $t=5$ hrs.

d) $G(t) - 100 = 0 \Rightarrow t = 4.923$

$A(0) = 500$

$\boxed{A(4.923) = 635.376 \text{ tons}}$

$A(8) = 525.55$

amount of gravel

$A(t) = 500 + \int_0^{t^*} (G(t) - 100)\,dt$

rate of change

$A' = G(t) - 100$