

Who's there? \_\_\_\_\_

### AP Calculus AB: Real-World FTC

Set-up the integral and use a calculator to find the answer to questions #1&2

1. If  $f'(x) = 3 + \frac{1}{x^2}$  and  $f(1) = 2$ , find  $f(2)$ . (answer:  $\frac{11}{2}$ )

$$2 + \int_1^2 3 + \frac{1}{x^2} dx = \frac{11}{2}$$

2. If  $y' = \frac{2}{x+3}$  and  $f(0) = 5$ , find  $f(3)$ . (answer:  $5 + \ln|4|$ )

$$5 + \int_0^3 \frac{2}{x+3} dx = 5 + 2 \ln|2| = 5 + \ln|4|$$

### You need a calculator for the rest of the worksheet

A tank contains 125 gallons of heating oil at time  $t = 0$ . During the time interval  $0 \leq t \leq 12$  hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

- (a) How many gallons of heating oil are pumped into the tank during the time interval  $0 \leq t \leq 12$  hours?

$$\int_0^{12} H(t) dt = \boxed{70.571 \text{ gallons}}$$

- (b) Is the level of heating oil in the tank rising or falling at time  $t = 6$  hours? Give a reason for your answer.

$H(6) = 5.395 \frac{\text{gal}}{\text{hr}}$  oil is falling because it is being pumped out faster than in.  
 $R(6) = 8.319 \frac{\text{gal}}{\text{hr}}$

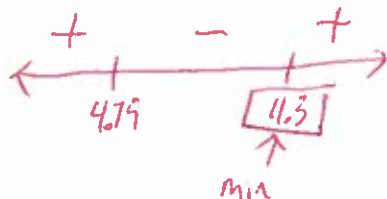
- (c) How many gallons of heating oil are in the tank at time  $t = 12$  hours?

$$125 + \int_0^{12} H(t) dt - \int_0^{12} R(t) dt = \boxed{122.026 \text{ gallons}}$$

- (d) At what time  $t$ , for  $0 \leq t \leq 12$ , is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

$$H(t) - R(t) = 0$$

$$t = 4.79 \text{ \& } 11.3 \quad \text{or}$$



Graph  $H(t) - R(t)$  and check where it goes from negative to positive values.

$$\boxed{t = 11.3}$$

Who's there? \_\_\_\_\_

**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

3. A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t$ , for  $0 \leq t \leq 5$ , is given by  $v(t) = \ln(t^2 - 3t + 3)$ . The particle is at position  $x = 8$  at time  $t = 0$ .
- Find the acceleration of the particle at time  $t = 4$ .
  - Find all times  $t$  in the open interval  $0 < t < 5$  at which the particle changes direction. During which time intervals, for  $0 \leq t \leq 5$ , does the particle travel to the left?
  - Find the position of the particle at time  $t = 2$ .
  - Find the average speed of the particle over the interval  $0 \leq t \leq 2$ .

2)  $v'(t) = a(t) = \frac{2t-3}{t^2-3t+3}$   
 $a(4) = \frac{2(4)-3}{(4)^2-3(4)+3} = \frac{5}{7}$

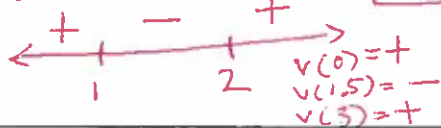
c)  $8 + \int_0^2 v(t) dt = 8.369$

d)  $\frac{\int_0^2 v(t) dt}{2} = 1.84$

b)  $v(t) = 0$

$\ln(t^2 - 3t + 3) = 0$   
 $t^2 - 3t + 3 = e^0$   
 $t^2 - 3t + 3 = 1$   
 $t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$   
 $t=2 \quad t=1$



Changes direction at  $t=1$  &  $t=2$

traveling to the left from  $[1, 2]$

Calc

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by  $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$ , where  $t$  is measured in hours and  $0 \leq t \leq 8$ . At the beginning of the workday ( $t = 0$ ), the plant has 500 tons of unprocessed gravel. During the hours of operation,  $0 \leq t \leq 8$ , the plant processes gravel at a constant rate of 100 tons per hour.
- Find  $G'(5)$ . Using correct units, interpret your answer in the context of the problem.
  - Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
  - Is the amount of unprocessed gravel at the plant increasing or decreasing at time  $t = 5$  hours? Show the work that leads to your answer.
  - What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

a)  $G'(5) = -24.588 \frac{\text{tons}}{\text{hr}^2}$

the acceleration of how fast gravel is arriving at  $t=5$

b)  $\int_0^8 G(t) dt = 825.551 \text{ tons}$

c)  $G(5) = 98.1408 \text{ tons/hr}$

and it's being processed at 100 tons/hr so the amount of gravel is decreasing at  $t=5$  hrs.

d)  $G(t) - 100 = 0 \Rightarrow t = 4.923$

$A(0) = 500$   
 $A(4.923) = 635.376 \text{ tons}$   
 $A(8) = 525.55$

amount of gravel  
 $A(t) = 500 + \int_0^t (G(t) - 100) dt$   
 rate of change  
 $A' = G(t) - 100$