Who's there? Key

AP Calculus AB: 8.7 Manipulating FTC

1. If $f'(x) = 3 + \frac{1}{x^2}$ and $f(1) = 2$, find $f(2)$. *(answer: $\frac{11}{2}$)*

$$2 + \int_1^2 f'(x)\,dx = 2 + \int_1^2 3 + \frac{1}{x^2}\,dx = 2 + \int_1^2 3 + x^{-2}\,dx$$

$$= 2 + \left[3x + \frac{x^{-1}}{-1}\right]_1^2 = 2 + \left[3x - \frac{1}{x}\right]_1^2 = 2 + \left[\left(3(2) - \frac{1}{2}\right) - \left(3(1) - \frac{1}{1}\right)\right] = \frac{11}{2}$$

2. If $y' = \frac{2}{x+3}$ and $f(0) = 5$, find $f(3)$. *(answer: 5+ln|4|)*

$u = x+3$
$du = dx$
$dx = du$

$$5 + \int_0^3 \frac{2}{x+3}\,dx = 5 + \int_3^6 \frac{2}{u}\,du = 5 + \left[2\ln|u|\right]_3^6 = 5 + \left[2\ln|6| - 2\ln|3|\right]$$

$$= 5 + \left[2\ln|2|\right] = 5 + \ln|4|$$

3. Calculator problem: Find $\cancel{g'}(4)$ of the function $g'(x) = \frac{x^2}{\ln(x)}$ if $g(2) = 3$

$$3 + \int_2^4 g'(x)\,dx =$$

4. A car's velocity is given by the function $v(t) = 3x^2 + 4x - 5$. If at t=0 the car's position is 3 meters.

(a) Find the position of the car after the first 2 seconds.

$$3 + \int_0^2 v(t)\,dt = 3 + \int_0^2 3t^2 + 4t - 5\,dt = 3 + \left[\frac{3t^3}{3} + \frac{4t^2}{2} - 5t\right]_0^2$$

$$3 + \left[t^3 + 2t^2 - 5t\right]_0^2 = 3 + (2)^3 + 2(2)^2 - 5(2) = \boxed{9}$$

(b) Find the function for acceleration.

$$v'(t) = a(t) = \boxed{6t + 4}$$

(c) Is the car speeding up or slowing down at time t=1?

$$v(1) = 3(1)^2 + 4(1) - 5 = +$$

$$a(1) = 6(1) + 4 = +$$

since both signs of v(t) and a(t) at t=1 are positive the car is speeding up.

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Calculator

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

   (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

   (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

   (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

   (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

a) $\boxed{G'(5) = -24.588 \text{ tons of gravel/hr}^2}$ ~~tons of gravel~~ the acceleration of how gravel is arriving to the plant at $t=5$ hours.

b) $\int_0^8 G(t)\,dt = \boxed{825.551 \text{ tons of gravel}}$

c) $G(5) = 98.141$ tons/hr

the amount is ~~increasing~~ decreasing at $t=5$ because the velocity of gravel arriving is is less than the velocity they are processing, 100 tons/hr.

d)

x	A(x)
0	500
8	525
4.924	635

Endpoints

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$A(x) = 500 + \int_0^x G(t) - 100\,dt$

$A'(x) = G(x) - 100 = 0$

$G(x) = 100$

$x = 4.924$

$A(0) = 500 + \int_0^0 G(x) - 100\,dx = 500$ tons

$A(8) = 525.551$ tons

$A(4.924) = 635.376$ tons

Max amount of gravel is 635.376 tons at $t = 4.924$ hours