

Sum up your name in this line: \_\_\_\_\_

Calculator  
Problem

|                             |      |      |      |      |      |
|-----------------------------|------|------|------|------|------|
| $t$ (minutes)               | 0    | 4    | 9    | 15   | 20   |
| $W(t)$ (degrees Fahrenheit) | 55.0 | 57.1 | 61.8 | 67.9 | 71.0 |

\* unequal bases

1. The temperature of water in a tub at time  $t$  is modeled by a strictly increasing, twice-differentiable function  $W$ , where  $W(t)$  is measured in degrees Fahrenheit and  $t$  is measured in minutes. At time  $t = 0$ , the temperature of the water is  $55^\circ\text{F}$ . The water is heated for 30 minutes, beginning at time  $t = 0$ . Values of  $W(t)$  at selected times  $t$  for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate  $W'(12)$ . Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Using trapezoid method of approximation, estimate  $\int_0^{20} W(t) dt$

(c) For  $0 \leq t \leq 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$d) \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = \frac{6.1}{6} = 1.017 \frac{\text{degrees F}}{\text{min}}$$

the temp is changing by  $1.017^\circ\text{F}$  every minute on avg between  $9 < t < 15$ .

$$b) \frac{4(55 + 57.1)}{2} + \frac{5(57.1 + 61.8)}{2} + \frac{6(61.8 + 67.9)}{2} + \frac{5(67.9 + 71)}{2} = 1257.8$$

$$c) \frac{4(55) + 5(57.1) + 6(61.8) + 5(67.9)}{20} = \frac{1215.8}{20} = 60.79^\circ\text{F}$$

It is an underestimation since the table is increasing.

Sum up your name in this line: Key

AP Calculus AB: 7.6 Real-World Riemann

|                             |   |     |    |      |      |
|-----------------------------|---|-----|----|------|------|
| $t$<br>(minutes)            | 0 | 2   | 5  | 8    | 12   |
| $v_A(t)$<br>(meters/minute) | 0 | 100 | 40 | -120 | -150 |

\* unequal bases

4. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function  $v_A(t)$ , where time  $t$  is measured in minutes. Selected values for  $v_A(t)$  are given in the table above.
- Find the average acceleration of train A over the interval  $2 \leq t \leq 8$ .
  - Do the data in the table support the conclusion that train A's velocity is  $-100$  meters per minute at some time  $t$  with  $5 < t < 8$ ? Give a reason for your answer.
  - At time  $t = 2$ , train A's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time  $t = 12$ . Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time  $t = 12$ .

a)  $a_{avg} = \frac{v_f - v_i}{t_f - t_i} = \frac{(-120) - (100)}{8 - 2} = \frac{-220}{6} = \frac{-110}{3} \text{ meters/min}^2$

b) yes,  $v(t) = -100 \text{ m/min}$  between  $5 < t < 8$  because by IVT there must be a value of  $-100$  since  $v_A(t)$  is differentiable.

c)  $300 + \int_2^{12} v_A(t) dt$  | trapezoid  $300 + \frac{3(100+40)}{2} + \frac{3(40-120)}{2} + \frac{4(-120-150)}{2} = -150 \text{ meters}$

|                    |   |     |     |      |      |      |      |
|--------------------|---|-----|-----|------|------|------|------|
| $t$<br>(minutes)   | 0 | 1   | 2   | 3    | 4    | 5    | 6    |
| $C(t)$<br>(ounces) | 0 | 5.3 | 8.8 | 11.2 | 12.8 | 13.8 | 14.5 |

\* equal bases

3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time  $t$ ,  $0 \leq t \leq 6$ , is given by a differentiable function  $C$ , where  $t$  is measured in minutes. Selected values of  $C(t)$ , measured in ounces, are given in the table above.
- Use the data in the table to approximate  $C'(3.5)$ . Show the computations that lead to your answer, and indicate units of measure.
  - Is there a time  $t$ ,  $2 \leq t \leq 4$ , at which  $C'(t) = 2$ ? Justify your answer.
  - Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of  $\frac{1}{6} \int_0^6 C(t) dt$ . Using correct units, explain the meaning of  $\frac{1}{6} \int_0^6 C(t) dt$  in the context of the problem.

a)  $\frac{f(4) - f(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = \frac{1.6}{1} = 1.6 \text{ ounces/min}$

b)  $\frac{f(4) - f(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = \frac{4}{2} = 2$  yes by MVT there is an IROC = AROC

c)  $\frac{(2(5.3) + 2(11.2) + 2(13.8))}{6}$

10.8 ounces the avg ounces dripping into the coffee maker