

ln(AME): Key

AP Calculus AB: Natural Log Derivatives & Integrals

Find the derivative of the following functions

1. $y = \ln(2x^4)$
 $y' = \frac{4 \cdot 2x^3}{2x^4} = \frac{4}{x}$

2. $y = \frac{\ln(x)}{5x}$
 $y' = \frac{5x(\frac{1}{x}) - \ln(x) \cdot 5}{(5x)^2}$
 $= \frac{5 - 5\ln(x)}{25x^2}$
 $\frac{1 - \ln(x)}{5x^2}$

3. $y = e^{-3x} \ln(6x^2)$
 $y' = e^{-3x} \left(\frac{12x}{6x^2} \right) + \ln(6x^2) - 3e^{-3x}$
 $= \frac{2e^{-3x}}{x} - 3e^{-3x} \ln(6x^2)$

Find the integral of the following functions

4. $\int \frac{-6}{6-8x} dx$ $u = 6-8x$
 $du = -8 dx$
 $dx = \frac{du}{-8}$
 $\int \frac{-6}{u} \frac{du}{-8}$
 $-\frac{3}{4} \int \frac{1}{u} du = -\frac{3}{4} \ln(u) + C$
 $= -\frac{3}{4} \ln(6-8x) + C$

5. $\int \frac{2x-7}{x^2-7x+3} dx$ $u = x^2-7x+3$
 $du = 2x-7 dx$
 $dx = \frac{du}{2x-7}$
 $\int \frac{2x-7}{u} \frac{du}{2x-7}$
 $\int \frac{1}{u} du$
 $\ln(u) + C = \ln(x^2-7x+3) + C$

6. $\int \frac{8x}{x^2-9} dx$ $u = x^2-9$
 $du = 2x dx$
 $dx = \frac{du}{2x}$
 $\int \frac{8x}{u} \frac{du}{2x}$
 $4 \ln(u) + C$
 $4 \ln(x^2-9) + C$

7. $\int \frac{\sin(x)}{2-6\cos(x)} dx$ $u = 2-6\cos(x)$
 $du = 6\sin(x) dx$
 $dx = \frac{du}{6\sin(x)}$
 $\int \frac{\sin(x)}{u} \frac{du}{6\sin(x)}$
 $\frac{1}{6} \ln(u) + C$
 $\frac{\ln(2-6\cos(x))}{6} + C$

8. $\int \frac{(\ln(x))^3}{x} dx$ $u = \ln(x)$
 $du = \frac{1}{x} dx$
 $dx = x du$
 $\int \frac{u^3}{x} x du$
 $\frac{u^4}{4} + C = \frac{\ln(x)^4}{4} + C = \frac{4 \ln(x)}{4} + C$
 $\ln(x) + C$

9. $\int \frac{x+2}{x^2+4x-7} dx$ $u = x^2+4x-7$
 $du = 2x+4 dx$
 $dx = \frac{du}{2x+4} = \frac{du}{2(x+2)}$
 $\int \frac{x+2}{u} \frac{du}{2(x+2)}$
 $\frac{1}{2} \ln(u) + C = \frac{\ln(x^2+4x-7)}{2} + C = \frac{1}{2} \ln(x^2+4x-7) + C$

10. $\int \frac{4x^3+5x-7}{\sqrt{x}} dx$
 $\int 4x^{5/2} + 5x^{1/2} - 7x^{-1/2} dx$
 $2 \cdot 4 \frac{x^{7/2}}{7} + \frac{5 \cdot 2 \cdot x^{3/2}}{3} - \frac{2 \cdot 7 \cdot x^{1/2}}{1} = \frac{8}{7} x^{7/2} + \frac{10}{3} x^{3/2} - 14\sqrt{x} + C$

11. $\int \frac{1}{\sqrt{4x-7}} dx = \int (4x-7)^{-1/2} dx$ $u = 4x-7$
 $du = 4 dx$
 $dx = \frac{du}{4}$
 $\int u^{-1/2} \frac{du}{4}$
 $\left(\frac{u^{1/2}}{1/2} \right) \cdot \frac{1}{4} = \frac{u^{1/2}}{2} = \frac{(4x-7)^{1/2}}{2} + C$