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AP Calculus AB: Exponential Integrals & Derivatives

Find the Derivatives of the following functions

1. $y = \frac{e^{-4x}}{16}$

$y' = \frac{-4e^{-4x}}{16} = \boxed{\frac{-e^{-4x}}{4}}$

2. $y = e^{x^2}\cos(5x)$

$y' = e^{x^2}(-\sin(5x)\cdot 5) + \cos(5x)\,2xe^{x^2}$

$= -5e^{x^2}\sin(5x) + 2xe^{x^2}\cos(5x)$

$= \boxed{e^{x^2}(-5\sin(5x) + 2x\cos(5x))}$

3. $y = e^{\tan(x)}$

$\boxed{y' = \sec^2(x)\,e^{\tan(x)}}$

Find the integrals of the following functions

4. $\int 4e^{2x}dx$

$u = 2x$, $du = 2dx$, $dx = \frac{du}{2}$

$\int 4e^u \frac{du}{2}$

$2e^u + C = \boxed{2e^{2x} + C}$

5. $\int \sec^2(x)e^{\tan(x)}dx$

$u = \tan(x)$, $du = \sec^2(x)dx$, $dx = \frac{du}{\sec^2(x)}$

$\int \sec^2(x)e^u \frac{du}{\sec^2(x)}$

$\int e^u du$

$e^u + C = \boxed{e^{\tan(x)} + C}$

6. $\int 12e^{6t}dt$

$u = 6t$, $du = 6\,dt$, $dt = \frac{du}{6}$

$\int 12e^u \frac{du}{6}$

$\int 2e^u du$

$2e^u + C = \boxed{2e^{6t} + C}$

7. Find all critical points of the graph $y = 3xe^{-x}$ and label them as max or mins.

$y' = 3x(-1e^{-x}) + e^{-x}(3)$

$y' = -3xe^{-x} + 3e^{-x}$

$y' = 3e^{-x}(-x + 1)$

$y' = 0$

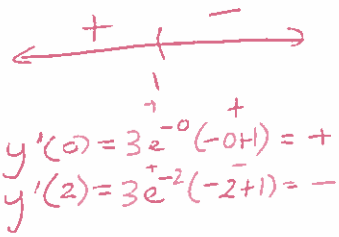
$3e^{-x}(-x+1) = 0$

$3e^{-x} = 0$ (no solution)

$-x + 1 = 0$

$-x = -1$

$x = 1$ ← max



$y'(0) = 3e^{-0}(-0+1) = +$

$y'(2) = 3e^{-2}(-2+1) = -$

8. The acceleration of a particle is given by $a(t) = 10e^{5t}$.

(a) Find the function for v(t) if v(0)=15 is given.

$v(t) = 2e^{5t} + C$

$15 = 2e^{5(0)} + C$

$15 = 2 + C$

$C = 13$

$\boxed{v(t) = e^{5t} + 13}$

$\int 10e^{5t}dt$

$u = 5t$, $du = 5\,dt$, $dt = \frac{du}{5}$

$\int 10e^u \frac{du}{5}$

$2e^u + C = 2e^{5t} + C$

(b) Is the particle slowing down or speeding up at t=5? Justify.

$v(5) = 2e^{5(5)} + 13 = +$

$a(5) = 10e^{5(5)} = +$

Speeding up at $t=5$ because $v(t)$ and $a(t)$ are positive at $t=5$

(c) Find the expression for position if at time t=0 the particle's position is x(t)=10

$\int e^{5t} + 13\,dt$

$u = 5t$, $du = 5dt$, $dt = \frac{du}{5}$

$\int e^u \frac{du}{5} + 13t + c$

$x(t) = \frac{2}{5}e^{5t} + 13t + C = \boxed{\frac{2}{5}e^{5t} + 13t + \frac{48}{5}}$

$10 = \frac{2}{5}e^{5(0)} + 13(0) + C$

$10 = \frac{2}{5} + C$

$C = 10 - \frac{2}{5} = \frac{50}{5} - \frac{2}{5} = \frac{48}{5}$