

U-Sub here: Key

AP Calculus AB: U-Substitution with Trigonometry

1.  $\int 6 \sin(3x) - 5 dx$   
 $\int 6 \sin(u) \frac{du}{3} - \int 5 dx$

$u = 3x$   
 $du = 3 dx$   
 $dx = \frac{du}{3}$

$-2 \cos(3x) - 5x + C$

2.  $-3 \int \cos\left(\frac{x}{3}\right) dx$   
 $\int -3 \cos(u) \frac{1}{3} du$

$u = \frac{x}{3}$   
 $du = \frac{1}{3} dx$   
 $dx = 3 du$

$\int -9 \cos(u) du$   
 $-9 \sin\left(\frac{x}{3}\right) + C$

3.  $\int \sin^2(x) \cos(x) dx$   
 $\int u^2 \cos(x) \frac{du}{\cos(x)}$   
 $\frac{u^3}{3} + C = \frac{\sin^3(x)}{3} + C$

$u = \sin(x)$   
 $du = \cos(x) dx$   
 $dx = \frac{du}{\cos(x)}$

$\frac{u^3}{3} + C = \frac{\sin^3(x)}{3} + C$

4.  $\int \frac{1}{2 \cos^2(x)} + 2x dx$   
 $\int \frac{\sec^2(x)}{2} + 2x dx$

$\frac{\tan(x)}{2} + \frac{2x^2}{2} + C$

$\frac{\tan(x)}{2} + x^2 + C$

5.  $\int \frac{\sin(x)}{\cos^2(x)} dx = \int \frac{\sin(x)}{\cos} \cdot \frac{1}{\cos} dx$

$= \int \tan(x) \sec(x) dx$

$\sec(x) + C$

6.  $\int 4 \sec^2(x) \tan^2(x) dx$

$u = \tan(x)$   
 $du = \sec^2(x) dx$   
 $dx = \frac{du}{\sec^2(x)}$

$\int 4 \sec^2(x) u^2 \frac{du}{\sec^2(x)}$

$\frac{4u^3}{3} + C = \frac{4 \tan^3(x)}{3} + C$

7.  $\int 10 \cos^4(x) \sin(x) dx$   
 $\int 10 u^4 \sin(x) \frac{du}{-\sin(x)}$   
 $-\frac{10u^5}{5} + C = -2 \cos^5(x) + C$

$u = \cos(x)$   
 $du = -\sin(x) dx$   
 $dx = \frac{du}{-\sin(x)}$

$-\frac{10u^5}{5} + C = -2 \cos^5(x) + C$

8.  $\int \frac{\csc\left(\frac{x}{2}\right) \cot\left(\frac{x}{2}\right)}{2} dx$

$u = \frac{x}{2}$   
 $du = \frac{1}{2} dx$   
 $dx = 2 du$

$\int \frac{\csc(u) \cot(u)}{2} \cdot 2 du$

$-\frac{\csc(u)}{2} + C = -\frac{\csc\left(\frac{x}{2}\right)}{2} + C$

9.  $6 \int \cos(2-3x) dx$   
 $\int 6 \cos(u) \frac{du}{-3}$   
 $-2 \sin(u) + C = -2 \sin(2-3x) + C$

$u = 2-3x$   
 $du = -3 dx$   
 $dx = \frac{du}{-3}$

$-2 \sin(u) + C = -2 \sin(2-3x) + C$

10.  $\int \csc(x) \cot(x) dx$

$-\csc(x) + C$