

Who you be? key

AP Calculus AB: Advanced Related Rates

1. A cylinder-like roll of paper towels has a height of 50 cm and is being unrolled. While it's being unrolled the radius is changing at 2 cm/sec. What is the rate of change of the surface area of the paper towel roll when the radius is 6 cm?

$h = 50$   
 $\frac{dr}{dt} = 2$   
 $r = 6$   
 $\frac{dSA}{dt} = ?$

(Ans:  $248\pi$  cm/sec)  $SA = 2\pi rh + 2\pi r^2$

$SA = 2\pi r(50) + 2\pi r^2$   
 $SA = 100\pi r + 2\pi r^2$

$\frac{dSA}{dt} = 100\pi \frac{dr}{dt} + 4\pi r \frac{dr}{dt}$

$\frac{dSA}{dt} = 100\pi(2) + 4\pi(6)(2) = 200\pi + 48\pi = 248\pi \frac{cm^2}{sec}$

2. A spherical balloon is being filled up with helium so that the radius has a rate of change of 3 centimeters per second. What is the rate at which the volume is changing at the instant the surface area is  $16\pi$  square centimeters. (Ans:  $48\pi$  cm<sup>3</sup>/sec)

$\frac{dr}{dt} = 3$   
 $\frac{dV}{dt} = ?$

$SA = 16\pi$   
 $r = 2$

$V = \frac{4}{3}\pi r^3$   $SA = 4\pi r^2$

$16\pi = 4\pi r^2$

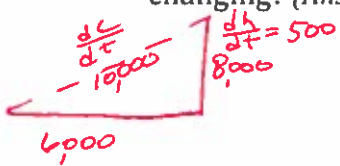
$4 = r^2$

$r = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(2)^2(3) = 48\pi \frac{cm^3}{sec}$

3. A camera is mounted 6,000 feet from the space shuttle launching pad. The camera needs to keep the shuttle in focus. If the shuttle is rising vertically at 500 ft/sec. When the shuttle is 8,000 ft high, how fast is the camera to shuttle distance changing? (Ans: 400 ft/sec)



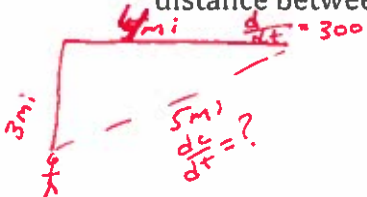
$a^2 + b^2 = c^2$   
 $6000^2 + b^2 = c^2$

$0 + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$

$2(8000)(500) = 2(10000) \frac{dc}{dt}$

$\frac{dc}{dt} = 400 \frac{ft}{sec}$

4. A person is trying to take a picture of an airplane; the airplane begins from 3 miles directly above him and is traveling at a rate of 300 miles per hour horizontally. When the airplane is exactly 5 miles away from the person, at what rate is the distance between the person and the airplane changing? (Ans: 240 mi/hr)



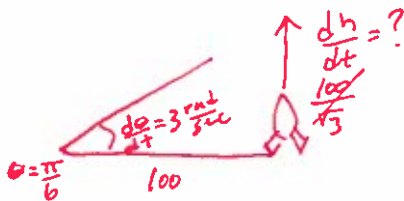
$a^2 + b^2 = c^2$

$0 + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$

$2(4)(300) = 2(5) \frac{dc}{dt}$

$\frac{dc}{dt} = 240 \frac{mi}{hr}$

5. A man is tracking a rocket with a telescope and wants to determine its velocity. The rocket is being launched from a launching pad that is 100 meters away. As the man follows the rocket with his telescope the angle is changing at a rate of 3 radians per second. What is the velocity of the rocket at the moment the telescope has an angle of  $\frac{\pi}{6}$ . (Ans: 400 m/s)



$\tan(\frac{\pi}{6}) = \frac{opp}{100}$

$\frac{\sqrt{1}}{\sqrt{3}} = \frac{h}{100}$

$h = \frac{100}{\sqrt{3}}$

not needed

$\tan(\theta) = \frac{h}{100}$

$\sec^2(\theta) \frac{d\theta}{dt} = \frac{1}{100} \frac{dh}{dt}$

$\sec^2(\frac{\pi}{6}) (3) = \frac{1}{100} \frac{dh}{dt}$

$(\frac{2}{\sqrt{3}})^2 (3) = \frac{1}{100} \frac{dh}{dt}$

$\frac{4}{3} (3) = \frac{1}{100} \frac{dh}{dt}$

$\frac{dh}{dt} = 400 \frac{m}{s}$