Hi, my name is Key

AP Calculus AB: 5.8 Intermediate Related Rates

1. A rock is dropped into a lake causing it to ripple in a circle. The radius of the outer ripple is increasing at a rate of 2 feet per second. When the radius is 6 feet, how is the rate of the area of the circle changing?

$\frac{dr}{dt} = 2\frac{ft}{sec}$

$r = 6$ ft

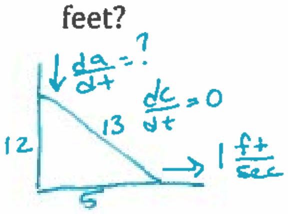
$\frac{dA}{dt} = ?$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(6)(2) = \boxed{24\pi \frac{ft^2}{sec}}$

2. A ladder that is 13 feet long is leaning against a wall of a building. The base of the ladder is being pulled away from the wall at a rate of 1 foot per second. How fast is the ladder falling down the wall when the base of the ladder to the wall measures 5 feet?



$a^2 + b^2 = 13^2$

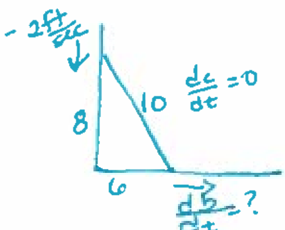
$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$2(12)\frac{da}{dt} + 2(5)(1) = 0$

$24\frac{da}{dt} + 10 = 0$

$\frac{da}{dt} = \frac{-10}{24} = \boxed{\frac{-5}{12} \frac{ft}{sec}}$

3. A ladder that is 10 feet long is leaning against a wall of a building. The ladder is falling down the wall at a rate of 2 feet per second. How fast is the ladder being pulled away from the wall when the base of the ladder to the wall measures 6 feet?



$a^2 + b^2 = 10^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$

$2(8)(-2) + 2(6)\frac{db}{dt} = 0$

$-32 + 12\frac{db}{dt} = 0$

$\frac{db}{dt} = \frac{32}{12} = \frac{16}{6} = \boxed{\frac{8}{3} \frac{ft}{sec}}$

4. A cone has a radius of 3 inches and the volume of the cone is increasing at a rate of 12π cubic inches per second. If the height of the cone is always 2 times the radius, find the rate of change of the radius. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$

$r = 3$

$\frac{dV}{dt} = 12\pi \frac{in^3}{sec}$

$h = 2r$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$

$12\pi = 2\pi(3)^2 \frac{dr}{dt}$

$12\pi = 18\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{12\pi}{18\pi} = \frac{6}{9} = \boxed{\frac{2}{3} \frac{in}{sec}}$

5. A spherical balloon is being filled with air at a rate of 2 cubic meters per minute. Find the rate at which the radius is changing when the volume is $\frac{32\pi}{3}$ cubic meters.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 2\frac{m^3}{min}$

$\frac{dr}{dt} = ?$

$V = \frac{32\pi}{3}$

$r = 2$

$\frac{32\pi}{3} = \frac{4\pi r^3}{3}$

$\frac{12\pi r^3}{12\pi} = \frac{96\pi}{12\pi}$

$r^3 = 8$

$r = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$2 = 4\pi(2)^2 \frac{dr}{dt}$

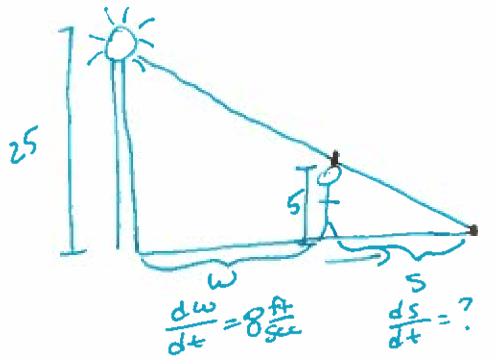
$2 = 16\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{2}{16\pi} = \boxed{\frac{1}{8\pi} \frac{m}{min}}$

Hi, my name is ______________________

6. A woman that is 5 feet tall is walking away from a 25-foot tall light post at a rate of 8 feet per second.

(a) What is the rate of change of the shadow's length? *(2 ft/sec)*



$$\frac{25}{w+s} = \frac{5}{s}$$

$$25s = 5w + 5s$$

$$20s = 5w$$

$$20\frac{ds}{dt} = 5\frac{dw}{dt}$$

$$20\frac{ds}{dt} = 5(8)$$

$$\frac{ds}{dt} = \frac{40}{20} = \boxed{2\ \frac{ft}{sec}}$$

(b) What is the rate of change from the tip of the shadow to the light post changing at? *(10 ft/sec)*

$$\frac{dw}{dt} + \frac{ds}{dt} = 8 + 2 = \boxed{10\ \frac{ft}{sec}}$$