

Nombre: Key

AP Calculus AB: 5.7 Beginner Related Rates

- 1) A circle has a radius of 10 inches but its radius is growing at a rate of 4 in/min. Find the rate at which the circumference is changing. (Ans:  $8\pi$  in/min)

$$r = 10 \quad \frac{dC}{dt} = ? \quad C = 2\pi r \quad \frac{dC}{dt} = 2\pi(4) = \boxed{8\pi \frac{\text{in}}{\text{min}}}$$
$$\frac{dr}{dt} = 4 \frac{\text{in}}{\text{min}} \quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

- 2) A circle has a radius of 15 inches but its radius is growing at a rate of 2 in./min. Find the rate at which the area is changing. (Ans:  $60\pi$  in<sup>2</sup>/min)

$$r = 15 \text{ in} \quad \frac{dA}{dt} = ? \quad A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(15)(2) = \boxed{60\pi \frac{\text{in}^2}{\text{min}}}$$
$$\frac{dr}{dt} = 2 \frac{\text{in}}{\text{sec}}$$

- 3) A spherical balloon is being inflated at a rate of 500 cubic centimeters per minute. How fast is the radius changing at the instant the radius is 20 cm? (Ans:  $\frac{5}{16\pi}$  cm/min)

$$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = ? \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{500}{1600\pi} \frac{\text{cm}}{\text{min}}$$
$$\frac{dV}{dt} = 500 \frac{\text{cm}^3}{\text{min}} \quad r = 20 \text{ cm} \quad 500 = 4\pi(20)^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{5}{16\pi} \frac{\text{cm}}{\text{min}}$$
$$500 = 1600\pi \frac{dr}{dt}$$

- 4) A rectangles width is increasing at a rate of 3 m/min and its area is increasing at a rate of 5 meters squared per min. What is the rate of change of the rectangles length at the instant the width is 5 meters and the length is 10 meters? (Ans:  $-5$  in/min)

$$\frac{dw}{dt} = 3 \frac{\text{m}}{\text{min}} \quad \frac{dL}{dt} = ? \quad A = Lw \quad 5 = 5 \frac{dL}{dt} + 30$$
$$\frac{dA}{dt} = 5 \frac{\text{m}^2}{\text{min}} \quad w = 5 \text{ m} \quad \frac{dA}{dt} = w \frac{dL}{dt} + L \frac{dw}{dt} \quad -25 = 5 \frac{dL}{dt}$$
$$l = 10 \text{ m} \quad 5 = (5) \frac{dL}{dt} + (10)(3) \quad \frac{dL}{dt} = -5 \frac{\text{m}}{\text{min}}$$

- 5) A circular based aquarium with a radius of 2 feet is being filled with water. If water is being poured into the tank at a rate of 8 cubic feet per minute, how fast is the water level rising?

$$\frac{dh}{dt} = ? \quad r = 2 \quad V = \pi r^2 h \quad 8 = \pi(2)^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = 8 \frac{\text{ft}^3}{\text{min}} \quad \frac{dr}{dt} = 0 \quad \frac{dV}{dt} = \cancel{h} 2\pi \frac{dr}{dt} + \pi r^2 \frac{dh}{dt} \quad 8 = 4\pi \frac{dh}{dt}$$
$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} \quad \frac{dh}{dt} = \frac{2}{\pi} \frac{\text{ft}}{\text{min}}$$

- 6) An oil spill has occurred and oil is spilling all over the street in a circular pattern. The area of the oil is changing at a rate of 2000 cubic meters per minute. At the instant the radius of the oil spill is 40 meters, what is the rate of change of the radius of the spill?

$$\frac{dA}{dt} = 2000 \frac{\text{m}^2}{\text{min}} \quad A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{2000}{80\pi} = \frac{25}{\pi} \frac{\text{m}}{\text{min}}$$
$$r = 40 \text{ m} \quad 2000 = 2\pi(40) \frac{dr}{dt} \quad \frac{dr}{dt} = ? \quad 2000 = 80\pi \frac{dr}{dt}$$