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AP Calculus AB: 5.3 Position, Velocity, Acceleration

1. A particle's position is given by the formula $s(t) = t^4 - 8t^2 + 9$ where $s(t)$ is measured in meters and t is measured in seconds for the interval $0 \leq t \leq 5$

a) Find the average velocity for the interval $1 \leq t \leq 2$

$$\frac{s(2) - s(1)}{2 - 1} = \frac{-7 - 2}{1} = \boxed{-9 \text{ meters/sec}}$$

b) Find the average acceleration from $t=0$ to $t=2$

$$v(t) = 4t^3 - 16t \quad \frac{v(2) - v(0)}{2 - 0} = \frac{0 - 0}{2} = \boxed{0 \text{ m/s}^2}$$

c) Find the instantaneous acceleration of the particle at $t=3$

$$a(t) = 12t^2 - 16 \quad a(3) = 12(3)^2 - 16 = \boxed{92 \text{ m/s}^2}$$

d) Find when the velocity is equal to zero.

$$4t^3 - 16t = 0 \quad 4t^2 - 16 = 0 \quad \boxed{t = 0, 2} \quad \text{No negative}$$

$$t(4t^2 - 16) = 0 \quad 4t^2 = 16$$

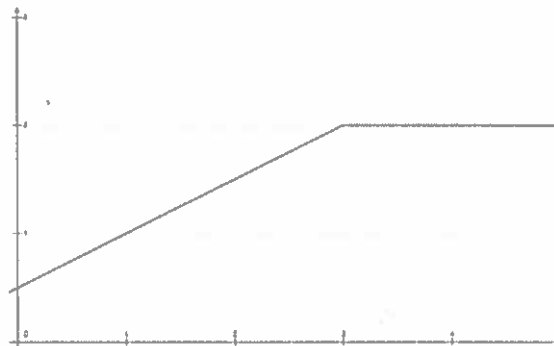
$$t = 0 \quad 4t^2 - 16 = 0 \quad t^2 = 4$$

$$t = \pm 2$$

2. A particle's position is given by the function $x(t) = \ln(t^2) \cdot f(t)$, the graph to the right is a graph of $f(t)$. Where $x(t)$ is measured in meters and t is measured in seconds.

a) Find the average velocity for $1 \leq t \leq 4$

$$\frac{x(4) - x(1)}{4 - 1} = \frac{\ln(16)(2) - \ln(1)(1)}{3} = \boxed{\frac{2 \ln(16)}{3} \text{ m/s}}$$



b) Find the instantaneous velocity at $t=1$

$$v(t) = f(t) \cdot \frac{2t}{t^2} + \ln(t^2) \cdot f'(t)$$

$$v(1) = (1) \frac{2(1)}{(1)^2} + \ln(1^2) \cdot \left(\frac{1}{2}\right) = \boxed{2 \text{ m/s}}$$

3. The table below gives a particle's velocity at given times. The function $v(t)$ is continuous and differentiable at all times.

t (sec)	0	2	5	6	9
v(t) (m/s)	3	7	8	5	10

a) How many times, if any, must $a(t)=0$? Justify.

$v'(t) = 0$ twice because of MVT and $v(t)$ is continuous and differentiable between $[5, 6]$ and $[6, 9]$

b) How many times, if any, must $v(t)=8$? Justify.

$v(t) = 8$ twice at $t=5$ and between $[6, 9]$ because of IVT.