Who had a lovely Thanksgiving? Key

AP Calculus AB: 5.2 Linear Approximation

1. Using the derivative and second derivative given by $\frac{dy}{dx} = xy^2$ and $\frac{d^2y}{dx^2} = y^2(2xy + 1)$ and the original point of $y(3) = -1$

$(-1)^2(2(3)(-1)+1) = +(-) = -$

a) Find the equation of the tangent line of the function above at x=3.

$y - y_1 = m(x - x_1)$

$y + 1 = 3(x - 3)$

$y = 3x - 10$

$y'(3) = (3)(-1)^2 = 3$

b) Using the tangent line at x=3 to estimate the value of $y(3.1)$, would this estimation be an over or under approximation? Justify.

$y'(3) = +$ ← inc

$y''(3) = -$ ← CD

over approximation since the graph f(x) is inc and CD at x=3

c) Using the first derivative $\frac{dy}{dx} = xy^2$, prove the second derivative $\frac{d^2y}{dx^2}$ above is correct.

$\frac{d^2y}{dx^2} = y^2(1) + x\,2y\frac{dy}{dx}$

$= y^2 + 2xy(xy^2)$

$= y^2 + 2x^2y^3$

$= y^2(1 + 2xy)$

(Calculator Allowed) 2. The amount of water inside of a city water tank is given by the function $W(t) = 5.329(0.871)^t$, where $W(t)$ is measured in gallons and t is measured in hours.

a) Find the average rate of change of $W(t)$ for the interval $0 \le t \le 15$

$\frac{W(15) - W(0)}{15 - 0} = \frac{.671 - 5.329}{15} = -.311 \frac{\text{gallons}}{\text{hr}}$

b) Find $W'(12)$ and explain what this value means.

$W'(12) = -0.140 \frac{\text{gallons}}{\text{hr}}$ this is the instantaneous rate of change of water into the tank at $t = 12$

c) Find the c-value guaranteed by the MVT where the IROC is equal to the AROC for the interval $0 \le t \le 15$

$W'(t) = -.311$ $t = 6.237$ hrs

d) Find the equation of the tangent line at t=12, and use it to approximate when $W(15)$.

$y - y_1 = m(x - x_1)$

$y - W(12) = W'(12)(x - 12)$

$y - 1.016 = -.140(x - 12)$

$y - 1.016 = -.140x + 1.68$

$y = -.140x + 2.696$

$y(15) = -.140(15) + 2.696$

$y(15) = .869$