Write your name parallel to this tangent line: Key

AP Calculus AB: 5.1 Equations of Tangent and Normal lines

1. Using the function $y = \frac{x+2}{x-3}$, find

$y' = \frac{(x-3)(1)-(x+2)(1)}{(x-3)^2} = \frac{-5}{(x-3)^2}$

a) the equation of a tangent line at x=2

$y - y_1 = m(x - x_1)$

$y + 4 = -5(x - 2)$

$y + 4 = -5x + 10$

$\boxed{y = -5x + 6}$

b) the equation of a normal line at x=2

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y + 4 = -\frac{1}{-5}(x - 2)$

$y + 4 = \frac{1}{5}(x - 2)$

$y + 4 = \frac{1}{5}x - \frac{2}{5}$

$\boxed{y = \frac{x}{5} - \frac{22}{5}}$

2. The function $T(t) = (2t-5)^3(t+1)$ measures the amount of trash inside of a dump at any given time, in days.

a) Find the average rate of change for trash per day at the dump for t=2 to t=3.

$\frac{T(3) - T(2)}{3 - 2} = \frac{4 - (-3)}{1} = \frac{7}{1} = \boxed{7 \frac{\text{trash}}{\text{day}}}$

b) Find the equation of a tangent line at t=3.

$T'(t) = (t+1)(3)(2t-5)^2 \cdot 2 + (2t-5)(1)$

$T'(3) = (4)(3)(1)^2 \cdot 2 + (1)(1)$

$24 + 1 = 25$

$y - y_1 = m(x - x_1)$

$y - 4 = 25(x - 3)$

$y - 4 = 25x - 75$

$\boxed{y = 25x - 71}$

3. Find the equation of the tangent line of the function $g(x) = e^{2x+5}$ where $g(x) = 1$

$\ln(1) = \cancel{\ln} e^{2x+5}$

$0 = 2x + 5$

$-5 = 2x$

$x = \frac{-5}{2}$

$g'(x) = 2e^{2x+5}$

$g'(-\frac{5}{2}) = 2e^{2(-\frac{5}{2})+5}$

$2e^0 = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x + \frac{5}{2})$

$y - 1 = 2x + 5$

$\boxed{y = 2x + 6}$

4. Write the equation of a tangent line for the function $f(x) = \ln(3x^2)$ at the point where $f(x)$ is parallel to $x - 2y = 6$.

$-2y = \cancel{-x} + 6$

$y = \frac{1}{2}x - 3$

$f' = \frac{6x}{3x^2} = \frac{2}{x}$

$\frac{2}{x} = \frac{1}{2}$

$4 = x$

$f(4) = \ln(3(4)^2)$

$\ln(48)$

$y - y_1 = \frac{1}{2}(x - x_1)$

$y - y_1 = \frac{1}{2}(x - 4)$

$y - \ln(48) = \frac{1}{2}x - 2$

$\boxed{y = \frac{1}{2}x - 2 + \ln(48)}$

5. Given the table, find the equation of the tangent line for the function $h(x)$ at x=2.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	3	-1	1	π

a) $h(x) = \ln(g(x))f(x)$

$y - y_1 = m(x - x_1)$

$y - y_1 = m(x - 2)$

$h(2) = \ln(1)(3)$

$= 0(3)$

$= 0$

$h'(x) = \frac{g'(x)}{g(x)} \cdot f(x) + \ln(g(x)) \cdot f'(x)$

$h'(2) = \frac{\pi}{1} \cdot (3) + \ln(1) \cdot (-1)$

$3\pi - \ln(1) = 3\pi$

$y = (3\pi)(x - 2)$

$\boxed{y = 3\pi x - 6\pi}$