

Will the real Key please stand up

AP Calculus AB: 4.3 Differentiability with variables

1. Find whether the function below is differentiable at their changing point or not. If it is not differentiable give the reason why.

$$f(x) = \begin{cases} x^3 + 5, & x < -2 \\ -3, & x > -2 \end{cases}$$

①  $\lim_{x \rightarrow -2} \text{Left } (-2)^3 + 5 = -3$   
 $\lim_{x \rightarrow -2} \text{Right } -3 = -3$

②  $f(-2)$  DNE  
 there is no equal sign

Not cont. or diff @  $x = -2$   
 b/c  $f(-2)$  DNE

2. Find the values a and b that will make the function  $f(x)$  differentiable at the value

$x=1.$

$$f(x) = \begin{cases} x^2 + 5, & x < 1 \\ bx^2 + ax, & x \geq 1 \end{cases}$$

cont

$$x^2 + 5 = bx^2 + ax$$

$$(1)^2 + 5 = b(1)^2 + a(1)$$

$$6 = b + a$$

$$b = 6 - a$$

diff

$$2x = 2bx + a$$

$$2(1) = 2b(1) + a$$

$$2 = 2b + a$$

sub

$$2 = 2(6 - a) + a$$

$$2 = 12 - 2a + a$$

$$2 = 12 - a$$

$$-10 = -a \quad a = 10$$

$$b = 6 - a$$

$$b = 6 - 10$$

$$b = -4$$

3. Find the values a and b that will make the function  $f(x)$  differentiable at the value

$x=2.$

$$f(x) = \begin{cases} ax^3 + 2, & x < 2 \\ x^2 + bx + 5, & x \geq 2 \end{cases}$$

$$a = \frac{1}{16} \quad b = -\frac{13}{4}$$

4. Find the values a and b that will make the function  $f(x)$  differentiable at the value

$x=-3.$

$$f(x) = \begin{cases} ax^2 + x - 8, & x \leq -3 \\ bx + 7, & x > -3 \end{cases}$$

$$a = -\frac{15}{9} \quad b = 11$$

5. Find the values a and b that will make the function  $f(x)$  differentiable at the value

$x=-1.$

$$f(x) = \begin{cases} 3ax^2 + 5ax, & x < -1 \\ b(2x + 3)^3 + 4, & x \geq -1 \end{cases}$$

cont

$$3ax^2 + 5ax = b(2x + 3)^3 + 4$$

$$3a(-1)^2 + 5a(-1) = b(2(-1) + 3)^3 + 4$$

$$3a - 5a = b + 4$$

$$-2a = b + 4$$

diff

$$6ax + 5a = 3b(2x + 3)^2 \cdot 2$$

$$6a(-1) + 5a = 3b(2(-1) + 3)^2 \cdot 2$$

$$-6a + 5a = 6b$$

$$-1a = 6b$$

$$a = -6b$$

sub

$$-2(-6b) = b + 4$$

$$12b = b + 4$$

$$11b = 4$$

$$b = \frac{4}{11}$$

$$a = -6\left(\frac{4}{11}\right) = -\frac{24}{11} = a$$