If you invented a theorem it would be Key Theorem

AP Calculus AB: 4.12 Extreme Value Theorem (EVT) & Intermediate Value Theorem (IVT)

1. Determine whether the function $f(x)$ below can be used for the Mean Value Theorem, Extreme Value Theorem, Intermediate Value Theorem or more than one of them. Justify.

$(2x+3)\cancel{(x-5)}$

$$f(x) = \begin{cases} \dfrac{2x^2 - 7x - 15}{\cancel{x-5}}, & x < 5 \\ x^2 - 8x + 28, & x \geq 5 \end{cases}$$

cont

1) $2(5)+3 = 13$
$(5)^2 - 8(5) + 28 = 13$ $\lim_{x \to 5} f(x) = 13$

2) $f(5) = 13$

3) $\lim_{x \to 5} f(x) = f(5)$

Can use IVT & EVT

diff

$L' = R'$

$2x - 8 = 2$

$2(5) - 8 = 2$

$2 = 2$

Can use MVT

2. The function below has variables that are unknown. Find what values for a & b would make the function possible to use with the mean value theorem.

$$f(x) = \begin{cases} x^2 + 8, & x < 1 \\ bx^2 + ax, & x \geq 1 \end{cases}$$

cont

$(1)^2 + 8 = b(1)^2 + a(1)$

$9 = b + a$

$a = 9 - b$

diff

$2x = 2bx + a$

$2(1) = 2b(1) + a$

$2 = 2b + a$

solving for variables

$2 = 2b + (9 - b)$

$2 = 2b + 9 - b$

$2 = b + 9$

$\boxed{b = -7}$

$a = 9 - b$

$a = 9 - (-7)$

$a = 9 + 7$

$\boxed{a = 16}$

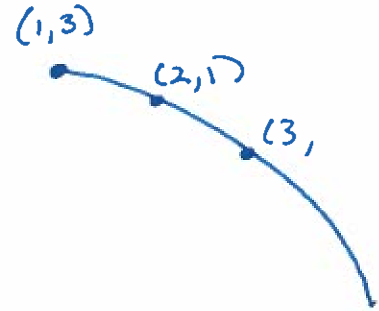
3. Let the function $f(x)$ be differentiable and have the points $f(-2) = -3$, $f(0) = 1$, and $f(1) = -4$. Which of the following is true, circle all that apply and write which theorem, if any, guarantees that statement next to it.

I. $f(x)$ has at least two x-intercepts IVT
II. $f(x)$ has a horizontal tangent line MVT
III. $f'(x) = 0$ at some point MVT
IV. For some value c, between 0<x<1, $f(c) = -2$ IVT

If you invented a theorem it would be ______________________________ Theorem

dec CD

4. If the function $f(x)$ is twice-differentiable and $f'(x) < 0$ *and* $f''(x) < 0$ for all $x > 1$ and the value $f(1) = 3$ *and* $f(2) = 1$ are points on the function $f(x)$. Circle all the values below that are possible.



I. ~~$f(3) = 5$~~

II. ~~$f(3) = 2$~~

III. ~~$f(3) = -1$~~

IV. $f(3) = -2$

V. $f(3) = -10$

5. Find the values of c, such that the mean value theorem is satisfied for the function $f(x) = (x+4)(x-1)^2$ for the interval $[-4,1]$.

$$f'(x) = \frac{f(1)-f(-4)}{1-(-4)}$$

$$\boxed{x = \frac{-7}{3}}$$

$$(x+4)\,2(x-1)'(1) + (x-1)^2(1) = \frac{0-0}{5}$$

$$(x-1)\left[2(x+4) + (x-1)\right] = 0$$

$$(x-1)\left[2x+8+x-1\right] = 0$$

$$(x-1)(3x+7) = 0$$

$$x = 1 \qquad x = \frac{-7}{3}$$

6. Find where the function's instantaneous rate of change is equal to it's average rate of change for the function $y = \sqrt{3x-6}$ on the interval [2,5].

$$(3x-6)^{1/2}$$

$$y' = \frac{f(5)-f(2)}{5-2}$$

$$\frac{1}{2}(3x-6)^{-1/2}(3) = \frac{3-0}{3}$$

$$\frac{3}{2\sqrt{3x-6}} = 1$$

$$3 = 2\sqrt{3x-6}$$

$$\frac{3}{2} = \sqrt{3x-6}$$

$$\frac{9}{4} = 3x-6$$

$$\frac{9}{4}+6 = 3x$$

$$\frac{9}{4}+\frac{24}{4} = 3x$$

$$\frac{33}{4} = 3x$$

$$\boxed{\frac{33}{12} = x}$$