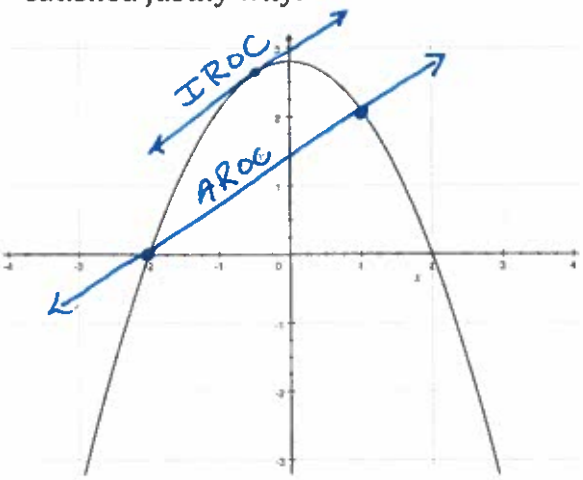
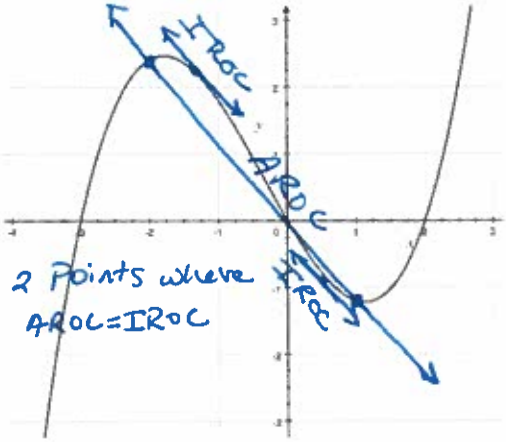
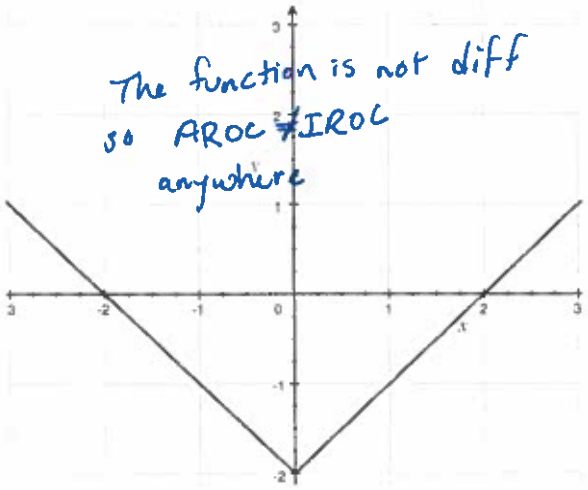
Who's mean? Key

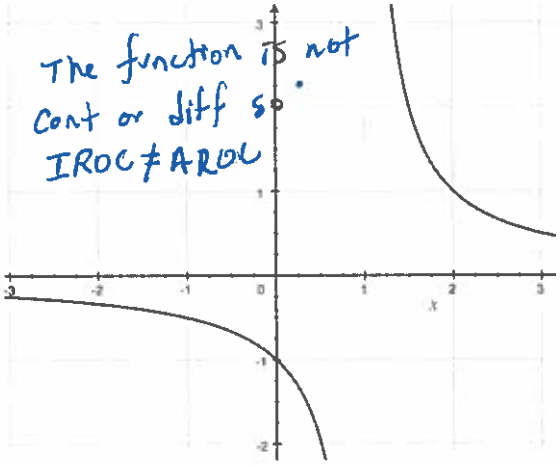
AP Calculus AB: 4.11 Mean Value Theorem

1. Using the graphs below, estimate the x-coordinates where the mean value theorem is satisfied for the interval [-2,1]. If there is nowhere that the MVT is satisfied justify why.









2. Find the x-value of c that satisfies the mean value theorem $f(x) = \sqrt{x^3} + 5$ on the interval [0,1].

$\sqrt{x^3} = x^{3/2}$

$f'(x) = \frac{f(1) - f(0)}{1 - 0}$

$\frac{3}{2}x^{1/2} = \frac{6-5}{1-0}$

$\frac{3\sqrt{x}}{2} = \frac{1}{1}$

$2 = 3\sqrt{x}$

$\frac{2}{3} = \sqrt{x}$

$\boxed{x = \frac{4}{9}}$

Who's mean?_____

3. Find the x-value where the function $f(x) = \frac{x-2}{x+2}$ instantaneous rate of change is equal to it's average rate of change on the interval [0,2].

$f'(x) = \frac{f(2)-f(0)}{2-0}$

$\frac{(x+2)(1)-(x-2)(1)}{(x+2)^2} = \frac{0-(-1)}{2}$

$\frac{x+2-x+2}{(x+2)^2} = \frac{1}{2}$

$\frac{4}{(x+2)^2} \times \frac{1}{2}$

$8 = (x+2)^2$

$\sqrt{8} = (x+2)$

$\sqrt{8} - 2 = x$

$\boxed{2\sqrt{2} - 2 = x}$

4. Using the piecewise function $f(x) = \begin{cases} 4x^2 - 9x, & x \le 2 \\ 7x - 16, & x > 2 \end{cases}$ answer the questions below.

a) Is the function $f(x)$ differentiable at x=2?

cont

1) $4(2)^2 - 9(2) = -2$ and $7(2) - 16 = -2$ $\Rightarrow \lim_{x \to 2} f(x) = -2$

2) $f(2) = -2$

3) $\lim_{x \to 2} f(x) = f(2)$

$f(x)$ is cont @ $x=2$

diff

1) $L' = R'$

$8x - 9 = 7$

$8(2) - 9 = 7$

$7 = 7$ ✓

$f(x)$ is differentiable @ $x=2$

b) What is the average rate of change for $f(x)$ over the interval [-2,1].

$\frac{f(1)-f(-2)}{1-(-2)} = \frac{(4(1)^2-9(1)) - (4(-2)^2-9(-2))}{3} = \frac{[4-9]-[16+18]}{3}$

$= \frac{-5-34}{3} = \frac{-39}{3} = \boxed{-13}$

c) Can you use the mean value theorem for the function $f(x)$? If you can then what c-value satisfies the MVT for the interval [0,2]

you can use the MVT on f(x) because it is differentiable over the interval [0,2]

$f'(x) = \frac{f(2)-f(0)}{2-0}$

$8x - 9 = \frac{[-2]-0}{2}$

$8x - 9 = \frac{-2}{2}$

$8x - 9 = -1$

$8x = 8$

$\boxed{x = 1}$