

What's your second name? Key Period: _____

AP Calculus: 3.17 Second Derivative of Implicit Differentiation

1. Find y'' of the function $x^2 + y^2 = 36$ at $(2, 1)$.

$$\frac{dy}{dx} = \frac{-x}{y} \leftarrow \text{quotient rule}$$

$$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2} = \frac{-y + x\frac{dy}{dx}}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} = \frac{-y - \frac{x^2}{y}}{y^2}$$

$$\text{@ } (2, 1) \quad \frac{-1 - \frac{(2)^2}{1}}{(1)^2} = \frac{-1 - 4}{1} = \boxed{-5}$$

2. Find $\frac{d^2y}{dx^2}$ of $x^2 + [xy] = 4$ at $(-1, 3)$
product

$$\frac{dy}{dx} = \frac{-2x - y}{x}$$

$$\frac{d^2y}{dx^2} = \frac{-x\left(\frac{-2x - y}{x}\right) + y}{x^2} = \frac{-2x - y + y}{x^2} = \frac{-2}{x}$$

$$\text{@ } (-1, 3) \quad \frac{d^2y}{dx^2} = \boxed{2}$$

3. Find the second derivative of $4[xy] - y = 25$ using implicit differentiation.
product

$$\frac{dy}{dx} = \frac{-4y}{4x - 1}$$

$$\frac{d^2y}{dx^2} = \frac{32y}{(4x - 1)^2}$$