

Will the real 16x please stand up.

Period: _____

AP Calculus: 3.16 Implicit Differentiation

1. Find y' of the function $4x^2 - y = 7xy^3$ ← product rule

$$8x - \frac{dy}{dx} = 7x(3y^2 \frac{dy}{dx}) + y^3(7)$$

$$8x - \frac{dy}{dx} = 21xy^2 \frac{dy}{dx} + 7y^3$$

$$8x - 7y^3 = 21xy^2 \frac{dy}{dx} + \frac{dy}{dx}$$

$$8x - 7y^3 = \frac{dy}{dx}(21xy^2 + 1)$$

$$\frac{8x - 7y^3}{21xy^2 + 1} = \frac{dy}{dx}$$

2. Find the derivative of $6y = y^2 - 5x^3$

$$6 \frac{dy}{dx} = 2y \frac{dy}{dx} - 15x^2$$

$$6 \frac{dy}{dx} - 2y \frac{dy}{dx} = -15x^2$$

$$\frac{dy}{dx}(6 - 2y) = -15x^2$$

$$\frac{dy}{dx} = \frac{-15x^2}{6 - 2y}$$

3. Find the instantaneous rate of change of the function $x^2 - 2xy + y^2 = 5$ at $(1, 2)$

$$2x - (2x \frac{dy}{dx} + y(2)) + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2y - 2x}{-2x + 2y} = \frac{2(2) - 2(1)}{-2(1) + 2(2)} = \frac{4 - 2}{-2 + 4} = \frac{2}{2} = \boxed{1}$$

4. Find $\frac{dy}{dx}$ of the function $25 - \tan(xy) = 4x$

chain and product rule

$$-\sec^2(xy)(x \frac{dy}{dx} + y(1)) = 4$$

$$-\sec^2(xy)[x \frac{dy}{dx} + y] = 4$$

$$-x \frac{dy}{dx} \sec^2(xy) - y \sec^2(xy) = 4$$

$$-x \frac{dy}{dx} \sec^2(xy) = 4 + y \sec^2(xy)$$

5. For the function $\frac{x+1}{x-1} = y^3$, find the derivative at $(3, 1)$

$$\frac{dy}{dx} = \frac{4 + y \sec^2(xy)}{-x \sec^2(xy)}$$

$$\frac{dy}{dx} = \frac{-2}{3y^2(x-1)^2}$$

$$\frac{dy}{dx} = \frac{-2}{3(1)^2(3-1)^2} = \frac{-2}{12} = \boxed{\frac{-1}{6}}$$