Who are you? Key

Calculus AB: 3.15 Mixing all the Rules (Chain, Product, Quotient and Power)

Find the instantaneous rate of change of each of the functions below.

a) $f(x) = 5x^4 + 6\sqrt{x} + 3$

$= 5x^4 + 6x^{1/2} + 3$

$\boxed{f'(x) = 20x^3 + \frac{3}{\sqrt{x}}}$

b) $f(x) = \frac{7}{2x^3} + \frac{4}{\sqrt[3]{x}} - 8$

$= \frac{7x^{-3}}{2} + 4x^{-1/3} - 8$

$f'(x) = \frac{-21x^{-4}}{2} - \frac{4x^{-4/3}}{3} = \boxed{\frac{-21}{2x^4} - \frac{4}{3\sqrt[3]{x^4}}}$

c) $f(x) = 7x^3 + 5\sin(3x)$

$\boxed{f'(x) = 21x^2 + 15\cos(3x)}$

d) $f(x) = (5x - 3)^4 + \ln(x^2)$

$\boxed{f'(x) = 20(5x-3)^3 + \frac{2}{x}}$

e) $f(x) = \csc(3x^2)$

$\boxed{f'(x) = -6x\csc(3x^2)\cot(3x^2)}$

Product

f) $f(x) = 3x / \sin(x)$

$\boxed{f'(x) = 3(x\cos(x) + \sin(x))}$

g) $f(x) = \frac{3x^2-4}{x+1}$ Hi Ho $d\,Hi = 6x$ $d\,Ho = 1$

$\boxed{f'(x) = \frac{3x^2+6x+4}{(x+1)^2}}$

h) $f(x) = \frac{\sin(x)}{\cos(x)}$

$f' = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$

$= \frac{1}{\cos^2(x)} = \boxed{\sec^2(x)}$

c) $f(x) = \frac{e^{4x}}{(2x+5)^2}$ Hi Ho $u = 4x$ $du = 4$ $d\,Hi = 4e^{4x}$

$d\,Ho = 2(2x+5)^1 \cdot 2 = 4(2x+5)$

$f'(x) = \frac{(2x+5)^2 4e^{4x} - e^{4x}(4(2x+5))}{((2x+5)^2)^2} = \frac{4e^{4x}\cancel{(2x+5)}(2x+5-1)}{(2x+5)^{\cancel{4}\,3}} = \boxed{\frac{4e^{4x}(2x+4)}{(2x+5)^3}}$

Product

1) $f(x) = 3x^2(x^2 - 5)^3$

$f'(x) = 3x^2(3(x^2-5)^2 \cdot 2x) + (x^2-5)^3 6x = 18x^3(x^2-5)^2 + 6x(x^2-5)^3$

$= 6x(x^2-5)^2(3x^2 + (x^2-5)) = \boxed{6x(x^2-5)^2(4x^2-5)}$

Who are you?________________________

2) Using the table below find the derivative of h(x) for each of the functions below at the value x=3.

x	f(x)	f '(x)	g(x)	g '(x)
3	4	-3	-2	7

a) $h(x) = f(x)g(x)$ *(Ans: 34)*

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$\boxed{34}$

b) $h(x) = f(x)^2$ *(Ans: -24)*

$h'(x) = 2f(x) \cdot f'(x)$

$\boxed{-24}$

c) $h(x) = \sqrt{f(x)}$ *(Ans: -3/2)*

$h'(x) = \dfrac{f'(x)}{2\sqrt{f(x)}}$

$\boxed{\dfrac{-3}{4}}$

d) $h(x) = f(x)^2 \cdot 3g(x)$ *(Ans: 480)*

$h'(x) = f(x)^2 \cdot 3g'(x) + 3g(x) \cdot 2f(x) \cdot f'(x)$

$\boxed{480}$

e) $h(x) = \frac{3f(x)}{-g(x)}$ *(Ans: 33/2)*

$h'(x) = \dfrac{-g(x)3f'(x) - 3f(x)(-g'(x))}{(g(x))^2}$

$\boxed{\dfrac{33}{2}}$

f) $h(x) = \arctan(f(x))$ *(Ans: -3/17)*

$h'(x) = \dfrac{f'(x)}{1+f(x)^2}$

$\boxed{\dfrac{-3}{17}}$