

Put your name double the times, that's the rule: Key

AP Calculus AB: 3.13 Double Chain Rule

Find $f'(x)$ for each of the following functions

1. $f(x) = \tan^4(3x^5 - x + 5)$

$$f'(x) = 4 \tan^3(3x^5 - x + 5) \sec^2(3x^5 - x + 5) (15x^4 - 1)$$

$$f' = (60x^4 - 4) \tan^3(3x^5 - x + 5) \sec^2(3x^5 - x + 5)$$

2. $f(x) = \sqrt{\csc(5x)} = \csc^{1/2}(5x)$

$$f' = \frac{1}{2} \csc^{-1/2}(5x) \cdot \csc(5x) \cot(5x) \cdot 5$$

$$= \frac{5 \csc(5x) \cot(5x)}{2 \sqrt{\csc(5x)}}$$

3. $f(x) = \sec^3(e^{4x^2})$

$$f' = 3 \sec^2(e^{4x^2}) \cdot \sec(e^{4x^2}) \tan(e^{4x^2}) \cdot 8xe^{4x^2}$$

$$= 24xe^{4x^2} \sec^3(e^{4x^2}) \cdot \tan(e^{4x^2})$$

~~4. $f(x) = \cos(\sqrt{\ln(x)})$~~

5. Evaluate $\lim_{h \rightarrow 0} \frac{4(x+h)^3 - \sin^3(6(x+h)^2) - (4x^3 - \sin^3(6x^2))}{h}$

$$12x^2 - 3 \sin^2(6x^2) \cdot \cos(6x^2) \cdot 12x$$

$$12x [x - 3 \sin^2(6x^2) \cos(6x^2)]$$

6. Find the instantaneous rate of change of the function $g(x) = \sin^2(4x)$ at $x = \frac{\pi}{3}$

$$g'(x) = 2 \sin(4x) \cdot \cos(4x) \cdot 4$$

$$8 \sin(4x) \cos(4x)$$

$$g'\left(\frac{\pi}{3}\right) = \left(\frac{8}{1}\right) \left(\sin\left(\frac{4\pi}{3}\right)\right) \left(\cos\left(\frac{4\pi}{3}\right)\right) = \left(\frac{8}{1}\right) \left(\frac{-\sqrt{3}}{2}\right) \left(\frac{-\sqrt{1}}{2}\right) = \frac{8\sqrt{3}}{4} = \boxed{2\sqrt{3}}$$