I solemnly swear Key will complete this worksheet

AP Calculus AB: 3.13 Derivatives of Inverse Functions

$g(1)=4$ $g(-6)=1$

1. Let $f(x)$ be a differentiable function and have the points $f(4)=1, f(1)=-6$, $f'(1)=-3, f'(-6)=4$ and let $g(x)=f^{-1}(x)$. Find $g'(-6)$.

$$g'(-6)=\frac{1}{f'(g(-6))}=\frac{1}{f'(1)}=\boxed{\frac{1}{-3}}$$

2. Let $f(x)$ be a differentiable function with an inverse function $g(x)$ and have the values $f(0)=1, f(1)=3, f'(3)=-3, and\ f'(1)=4$. Find $g'(3)$.

$$g'(3)=\frac{1}{4}$$

x	-2	-1	0	1	2
f(x)	4	2	-6	0	7
h(x)	-1	-5	0	2	9
f'(x)	3	9	4	-2	-7

(Hint: if this table confuses you then write out the values like the problems above)

3. Let $f(x)$ be a differentiable function and $g(x)=f^{-1}(x)$. Using the table above answer the questions below.

$f(-2)=4$ $g(4)=-2$
$f(-1)=2$ $g(2)=-1$
$f(0)=-6$ ⋮
$f(1)=0$
$f(2)=7$

a. Find $g'(4)$

$$g'(4)=\frac{1}{f'(g(4))}=\frac{1}{f'(-2)}=\boxed{\frac{1}{3}}$$

b. Find $g'(0)$

$$\boxed{-\frac{1}{2}}$$

c. Calculate $g'(2)$

$$\boxed{\frac{1}{9}}$$

d. Find the derivative of $\underbrace{2f(x)h(x)}_{\text{product}}$ at $x=1$

$$\boxed{-8}$$

e. Find $\frac{d}{dx}\left(\frac{3f(x)+2}{g(x)}\right)$ at the point $x=0$

$$\boxed{4}$$

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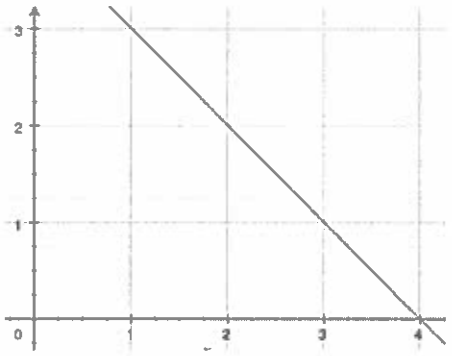
4. The graph of $f(x)$ is located to the right. You also know that $f(x)$ is the inverse of $g(x)$. Find the derivative of each of the following functions below using the graph to the right.



$f(1)=3$ $g(3)=1$
$f(2)=2$ $g(2)=2$
$f(3)=1$ $g(1)=3$
$f(4)=0$ $g(0)=4$

a) $y = f(x)^4 \text{ at } x = 3$

$y' = 4f(x)^3 \cdot f'(x)$

$y'(3) = 4(f(3))^3 f'(3) = 4(1)^3 \cdot (-1) = \boxed{-4}$

b) $y = \sqrt{f(x)}$ at $x = 0$

$y = (f(x))^{1/2}$

$y' = \frac{1}{2} f(x)^{-1/2} \cdot f'(x) = \frac{f'(x)}{2\sqrt{f(x)}}$

$y'(0) = \frac{f'(0)}{2\sqrt{f(0)}} = \frac{-1}{2\sqrt{4}} = \boxed{\frac{-1}{4}}$

c) Find $\frac{d}{dx}\left(\frac{3f(x)+2}{x^2}\right)$ at the point $x = 1$

$y' = \frac{x^2(3f'(x)) - (3f(x)+2)(2x)}{x^4}$

$\boxed{y'(1) = -25}$

d) Find the derivative of $g(x)$ at x=3 (Remember: g(x) is the inverse of f(x))

$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \boxed{-1}$

e) Find the instantaneous rate of change for $g(x)$ at x=4

$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{f'(g(4))} = \frac{1}{f'(0)} = \frac{1}{-1} = \boxed{-1}$