Who loves Polar functions????? Key

AP Calculus BC: 13.3 Polar Derivatives

Convert the following polar equations into parametric equations.

1. $r = 5\sin(\theta)$

$y = 5\sin^2(\theta)$

$x = 5\sin(\theta)\cos(\theta)$

2. $r = \frac{\cos(\theta)}{3}$

$y = \frac{\sin(\theta)\cos(\theta)}{3}$

$x = \frac{\cos^2(\theta)}{3}$

3. Find $\frac{dy}{dx}$ for the function $r = 3\sin(\theta)$.

$y = 3\sin^2(\theta)$

$x = 3\sin(\theta)\cos(\theta)$

$\frac{dy}{d\theta} = 6\sin(\theta)\cdot\cos(\theta)$ $\quad \frac{dx}{d\theta} = -3\sin^2(\theta) + 3\cos^2(\theta)$

$$\boxed{\frac{dy}{dx} = \frac{6\sin(\theta)\cos(\theta)}{-3\sin^2(\theta) + 3\cos^2(\theta)}}$$

i) at $\theta = \frac{\pi}{4}$

$$\frac{6\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{-3\left(\frac{\sqrt{2}}{2}\right)^2 + 3\left(\frac{\sqrt{2}}{2}\right)^2} = \frac{3}{-\frac{3}{4} + \frac{3}{4}} = \boxed{\text{und}}$$

ii) at the polar point $(3, \frac{\pi}{2})$

$$\frac{6(1)(0)}{-3(1)^2 + 3(0)^2} = \frac{0}{-3} = \boxed{0}$$

4. Find the slope of the line tangent to the graph $r = 2 - 3\cos(\theta)$ at $\theta = \frac{\pi}{2}$

$y = 2\sin(\theta) - 3\cos(\theta)\sin(\theta)$

$x = 2\cos(\theta) - 2\cos^2(\theta)$

$\frac{dy}{d\theta} = 2\cos(\theta) - 3\cos^2(\theta) + 3\sin^2(\theta)$

$\frac{dx}{d\theta} = -2\sin(\theta) + 4\cos(\theta)\sin(\theta)$

$$\frac{dy}{dx} = \frac{2\cos(\theta) - 3\cos^2(\theta) + 3\sin^2(\theta)}{-2\sin(\theta) + 4\cos(\theta)\sin(\theta)}$$

$$\frac{3(1)^2}{-2(1)} = \boxed{\frac{3}{-2}}$$

5. (Calculator) Using the equation of the tangent line at $\theta = \frac{\pi}{6}$ estimate when the function $r = -2\cos(\theta)$, is equal to 3.

$y = -2\cos(\theta)\sin(\theta)$

$x = -2\cos^2(\theta)$

$\frac{dy}{d\theta} = -2\cos^2(\theta) + 2\sin^2(\theta)$

$\frac{dx}{d\theta} = +4\cos(\theta)\sin(\theta)$

$$\frac{dy}{dx} = \frac{-2\cos^2(\theta) + 2\sin^2(\theta)}{4\cos(\theta)\sin(\theta)} = \frac{-1}{\sqrt{3}}$$

$$y + \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}}\left(x + \frac{3}{2}\right) \Rightarrow y = -\frac{1}{\sqrt{3}}\left(x + \frac{3}{2}\right) - \frac{\sqrt{3}}{2}$$

$$3 = \frac{-1}{\sqrt{3}}\left(x + \frac{3}{2}\right) - \frac{\sqrt{3}}{2}$$

$\boxed{x = -8.196}$

6. Find the horizontal and vertical tangent lines of $r = 2\cos(\theta)$ in terms of θ.

$y = 2\cos(\theta)\sin(\theta)$

$x = 2\cos^2(\theta)$

$\frac{dy}{d\theta} = 2\cos^2(\theta) - 2\sin^2(\theta)$

$\frac{dx}{d\theta} = 4\cos(\theta)\sin(\theta)$

$$\frac{dy}{dx} = \frac{2\cos^2(\theta) - 2\sin^2(\theta)}{4\cos(\theta)\sin(\theta)} = 0 \quad \text{HTL}$$

$$4\cos(\theta)\sin(\theta) = 0 \quad \text{VTL}$$

$2(\cos^2(\theta) - \sin^2(\theta)) = 0$

$\cos^2(\theta) = \sin^2(\theta)$

$\cos(\theta) = \sin(\theta)$

$\theta = \pi/4, \frac{3\pi}{4}$

HTL

$\cos(\theta) = 0 \quad \sin(\theta) = 0$

$\theta = \pi/2 \qquad \theta = 0, \pi$

VTL