Polar Name: Key

AP Calculus BC: 13.1 Intro to Polar Formulas

Convert the following rectangular coordinates to polar coordinates

1. (2,4)

$\sqrt{2^2+4^2} = 4.47$

$\tan^{-1}\left(\frac{4}{2}\right) = 1.107$

$(4.47, 1.107)$

2. (-3,2)

$\sqrt{(-3)^2+(2)^2} = \sqrt{13}$

$\tan^{-1}\left(\frac{2}{3}\right) = .588$

$(\sqrt{13}, 2.554)$

3. (5,-1)

$\sqrt{5^2+(-1)^2} = \sqrt{26}$

$\tan^{-1}\left(\frac{1}{5}\right) = .197$

$(\sqrt{26}, 6.086)$

Convert the following polar coordinates to rectangular coordinates

4. $(3, \frac{\pi}{4})$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

5. $(4, \frac{2\pi}{3})$

$(2\sqrt{3}, -2)$

6. $(1, \frac{7\pi}{6})$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Convert the following rectangular formulas to polar formulas

7. $y = 5x - 2$

$r\sin(\theta) = 5(r\cos(\theta)) - 2$

$r\sin(\theta) - 5r\cos(\theta) = -2$

$r(\sin(\theta) - 5\cos(\theta)) = -2$

$r = \frac{-2}{\sin(\theta) - 5\cos(\theta)}$

8. $3x - y = 2$

$3r\cos(\theta) - r\sin(\theta) = 2$

$r(3\cos(\theta) - \sin(\theta)) = 2$

$r = \frac{2}{3\cos(\theta) - \sin(\theta)}$

9. $y^2 + x^2 = 9$

$(r\sin(\theta))^2 + (r\cos(\theta))^2 = 9$

$r^2(\sin^2(\theta) + \cos^2(\theta)) = 9$

$r^2 = \frac{9}{\sin^2(\theta) + \cos^2(\theta)} \Rightarrow r = \frac{3}{\sqrt{1}} = \boxed{3}$

10. $y^2 = xy + 3$

$r^2\sin^2(\theta) = r\cos(\theta)\, r\sin(\theta) + 3$

$r^2\sin^2(\theta) - r^2\cos(\theta)\sin(\theta) = 3$

$r^2 = \frac{9}{\sin^2(\theta) - \cos(\theta)\sin(\theta)} = \boxed{\frac{3}{\sqrt{\sin^2(\theta) - \cos(\theta)\sin(\theta)}}}$

Convert the following polar formulas to rectangular formulas

11. $r = \frac{3}{2\sin(\theta)}$

$r\sin(\theta) = \frac{3}{2}$

$\boxed{y = \frac{3}{2}}$

12. $r = \frac{2\sin(\theta)+1}{\cos(\theta)}$

$r\cos(\theta) = 2r\sin(\theta) + 1$

$x = 2y + 1$

$x - 1 = 2y$

$\boxed{y = \frac{x-1}{2}}$

13. $r = \frac{7}{\sin(\theta) - \cos(\theta)}$

$r(\sin(\theta) - \cos(\theta)) = 7$

$r\sin(\theta) - r\cos(\theta) = 7$

$y - x = 7$

$\boxed{y = x + 7}$

14. $r = 2\csc(\theta)$

$r = \frac{2}{\sin(\theta)}$

$r\sin(\theta) = 2$

$\boxed{y = 2}$