_____ is so cool it hurts!

AP Calculus BC: 12.6 Vectors Magnitude & Distance

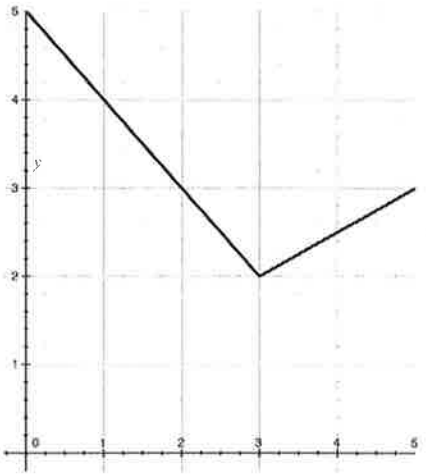
1. (Calculator) A parametric equation, $(x(t), y(t))$, defines the path of motion of a particle. If the velocity vector is given by $v(t) = (\ln(t - 1.2) + 3,\ sin^2(t))$.
   a) Find the distance traveled by the particle on the t interval [2,4]

$$\int_0^4 \sqrt{(\ln(t-1.2)+3)^2 + (\sin^2(t))^2}\,dt = \boxed{7.061}$$

   b) Find the speed of the particle at t=3

$$\sqrt{x'(3) + y'(3)} = \boxed{3.588}$$

2. (Calculator) A parametric equation, $(x(t), y(t))$, shows the motion for a particle in terms time for $0 \le t \le 5$. A graph of $y(t)$ is given and the velocity of the horizontal component of the particle is give by $\frac{dx}{dt} = 3t + \sin(t)$. At $t = 0$ the position of the particle is (2,5).



   a) Find the position of the particle at $t = 1$.

$$2 + \int_0^1 3t + \sin(t)\,dt = 3.509 \qquad (3.509, 4)$$

   b) Find the velocity vector for the particle at t=2.

$$x'(2) = \qquad (3.017, -1)$$

   c) Find the distance traveled by the particle from t=0 to t=3.

$$\int_0^3 \sqrt{x'(t)^2 + y'(t)^2}\,dt = \boxed{14.142}$$

($y'(t) = -1$)

   d) Find the speed of the particle at t=4.

$$\sqrt{x'(4)^2 + y'(4)^2} = \boxed{11.254}$$

   e) Find the slope of the line tangent to the motion of the particle at t=4.

$$\frac{\frac{1}{2}}{3t + \sin(t)} = \frac{3t + \sin(t)}{2} \longrightarrow \boxed{5.622}$$

   f) Describe the horizontal motion of the particle at t=3.

$x'(3) = 3(3) + \sin(3) = \boxed{9.141}$ The particle is moving to the right at $t = 3$.

   g) Find the total distance traveled by the particle over the entire $0 \le t \le 5$ time interval.

$$\int_0^3 \sqrt{x'(t)^2 + y'(t)^2}\,dt + \int_3^5 \sqrt{x'(t)^2 + y'(t)^2}\,dt = \boxed{38.720}$$

($y'(t) = -1$; $y'(t) = \frac{1}{2}$)