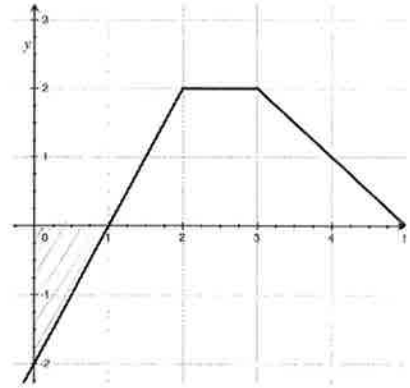
The position of your name goes there→ Key

AP Calculus BC: 12.5 Vector PVA

1. A particle is moving on an xy-plane where it's position is given by the parametric equations $(x(t), y(t))$, where $\frac{dx}{dt} = \frac{1}{1+t^2}$ and a graph of $\frac{dy}{dt}$ is provided. If at t=0 the particle's position is $(-2,4)$,



a) Find the position of the particle at t=1.

$x(t) = -2 + \int_0^1 \frac{1}{1+t^2} dt = -1.215$

$y(t) = 4 + \int_0^1 \frac{dy}{dt} dt = 4 + \frac{(1)(-2)}{2} = 3$

$\boxed{(-1.215, 3)}$

b) Find the velocity vector of the particle at t=1.

$\boxed{\left(\frac{1}{2}, 0\right)}$

c) Find the acceleration vector of the particle at t=1

$\left(-\frac{1}{2}, 2\right)$

$\frac{d^2x}{dt^2} = \frac{(1+t^2)-(1)(2t)}{(1+t^2)^2}$

d) Find $\frac{d^2y}{dx^2}$ at $t = 1$.

$\frac{d^2y}{dx^2} = \frac{2t}{\frac{1}{1+t^2}} = 2(1+t^2)$

$2(1+(1)^2) = \boxed{4}$

e) Describe the horizontal movement of the particle at t=2.

$\frac{1}{1+(2)^2} = \frac{1}{5}$

x(t) is positive so that means the particle is moving to the right at t=2.

f) Give the interval when the vertical movement of the particle is downward, in terms of t. Justify.

[0, 1] because $\frac{dy}{dt}$, which represents vertical motion, is negative.

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2. (Calculator) For $t \geq 0$, a particles motion is given by $(x(t), y(t))$ with a velocity vector of $v(t) = (-e^{0.3t}, \ln(t^2 + t))$. The position of the particle is $(2,5)$ when $t = 1$.
   a) Find the y-coordinate of position when $t = 3$.

   $5 + \int_1^3 \ln(t^2+t)\,dt = \boxed{8.455}$

   b) Find the acceleration vector for the particle at $t = 2$.

   $\boxed{\langle -0.547, 0.833 \rangle}$

   c) Find when the particles vertical motion is downward, in terms of t.

   graph v(t)

   $\boxed{(0, 0.618]}$

   d) At what time, t, does the particles path of motion have a tangent line equal to 0.5?

   $\frac{dy}{dx} = .5 \qquad \frac{\ln(t^2+t)}{-e^{.3t}} = .5 \qquad \boxed{t = .405}$

3. For $t \geq 0$, a particles motion is described by the parametric equations $(x(t), y(t))$. Where $x(t) = t + \ln(t - 3)$ and the slope of $y(t)$ is given by the formula $\frac{dy}{dt} = t^2 - 7t + 10$. At $t = 0$ the vertical position is 3.
   a) Find the interval, in terms of t, when the particle has horizontal motion to the right.

   $\frac{dx}{dt} = 1 + \frac{1}{t-3} \rightarrow 1 + \frac{1}{t-3} = 0 \qquad \frac{t-3}{t-3} + \frac{1}{t-3} = 0 \rightarrow \frac{t-2}{t-3} = 0 \qquad t = 3 \quad t = 2$

   $+ \quad - \quad +$

   $2 \quad 3$

   $\boxed{[0,2], [3, \infty)}$

   b) Find the slope of the tangent line to the path of the particle at t=9.

   $\frac{dy}{dx} = \frac{t^2 - 7t + 10}{1 + \frac{1}{t-3}} = \frac{t^2 - 7t + 10}{\frac{t-2}{t-3}} = \frac{(t-3)\cancel{(t-2)}(t-5)}{\cancel{t-2}} = (t-3)(t-5)$

   $(9-3)(9-5) = 6(4) = \boxed{24}$

   c) Find $\frac{d^2y}{dx^2}$ for the particles motion at t=4.

   $\frac{d^2y}{dx^2} = \frac{(t-3)+(t-5)}{1 + \frac{1}{t-3}} = \frac{2t-8}{\frac{t-2}{t-3}} = \frac{(2t-8)(t-3)}{t-2} \rightarrow \frac{(0)(1)}{2} = \boxed{0}$