How many seconds does it take to draw the curve of your name? Key

AP Calculus BC: 12.4 Length of a Curve for Parametric Equations

Calculator may be used throughout this worksheet

1. Find the length of the parametric curve given the equations $x = t^2 + 3t$ and $y = \ln(5t+1)$ on the interval [0,6]. *(Ans: 54.563)*

$$\int_0^6 \sqrt{(2t+3)^2+\left(\frac{5}{5t+1}\right)^2}\,dt = 54.563$$

2. Find the length of the parametric curve given the equations $x = \arccos(2t)$ and $y = 5t^2$ on the interval [0,0.5]. *(Ans: 2.069)*

$$\int_0^{.5} \sqrt{\left(\frac{-2}{\sqrt{1-4t^2}}\right)^2+(10t)^2}\,dt = 2.069$$

3. A parametric equation is given the equations $(x(t), y(t))$ for the interval [0,6] and it is known that $\frac{dx}{dt} = \sqrt{t-1}$ and $\frac{dy}{dt} = e^{\frac{t}{2}} - 5$.

a) When are the parametric equations decreasing on the interval, in terms of t.

$$\frac{dy}{dx} = \frac{e^{t/2}-5}{\sqrt{t-1}}$$

$e^{t/2}-5=0 \qquad t=\ln(25)$

$\sqrt{t-1}=0 \qquad t=1$

und

1 ln(25)

b) Find the $\frac{d^2y}{dx^2}$ at $t = 2$

$\boxed{5/2}$

calculate fn calc.

c) Find the length of the parametric curve for the t-interval [0,6].

$$\int_0^6 \sqrt{(\sqrt{t-1})^2+(e^{t/2}-5)^2}\,dt = \boxed{26.456}$$

d) At what t-value, on the interval [0,6], does $y(t)$ have a minimum value? If $y(t)$ has a point $y(0) = 3$, find the y-value at the t-value where $y(t)$ has a minimum.

$e^{t/2}-5=0$

$e^{t/2}=5$

$t/2=\ln(5)$

$t=2\ln(5)=\ln(25)$

ln(25)

$$3+\int_0^{\ln(25)} e^{t/2}-5\,dt$$

$\boxed{-5.094}$