x(first name) and y(last name)= Key

AP Calculus BC: 12.2 Derivatives of Parametric Equations

1. For the parametric equation $x(t) = 6t^3,\ y(t) = 9t - 2$ find $\frac{dy}{dx}$ for t=-2.

$\frac{dy}{dx} = \frac{9}{18t^2} = \frac{1}{2t^2} \rightarrow \frac{1}{2(-2)^2} = \boxed{\frac{1}{8}}$

2. For the parametric equation $x(t) = 2t,\ y(t) = 3t^2$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the point (4,12). $4 = 2t$, $t = 2$

$\frac{dy}{dx} = \frac{6t}{2} = 3t \rightarrow 3(2) = \boxed{6}$ $\quad \frac{d^2y}{dx^2} = \boxed{\frac{3}{2}}$

3. For the parametric equation $x(t) = \sin(t)$ and $\frac{dy}{dt} = \cot(t)$ find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{6}$.

$\frac{d^2y}{dx^2} = \frac{-\csc^2(t)}{\cos(t)} \rightarrow \frac{-\left(\frac{2}{\sqrt{1}}\right)^2}{\frac{\sqrt{3}}{2}} = \frac{\frac{-4}{1}}{\frac{\sqrt{3}}{2}} = \frac{-8}{\sqrt{3}} = \boxed{\frac{-8\sqrt{3}}{3}}$

4. A parametric curve is defined by the equations $x = 6\sqrt{t^3}$ and $y = t^2 - 2t$, find the equation of a tangent line to the curve at $t = 1$ $\quad 6t^{3/2}$

$\frac{dx}{dt} = \frac{18}{2}t^{1/2} = 9\sqrt{t}$

$\frac{dy}{dx} = \frac{2t-2}{9\sqrt{t}}$

$\frac{2(1)-2}{9\sqrt{1}} = \frac{0}{9} = 0$

$y + 1 = 0(x - 6)$

$\boxed{y = -1}$

5. A parametric curve is defined by the equations $x = \sqrt{t^3} + 2$ and $y = \sqrt{t}$, find the equation of a tangent line to the curve at the point (10,2) and use the tangent line to approximate when the parametric equation is equal to 3, in terms of x. $\quad x = t^{3/2} + 2$

$y - 2 = \frac{1}{12}(x - 10)$

$3 - 2 = \frac{1}{12}(x - 10)$

$1 = \frac{1}{12}(x - 10)$

$12 = x - 10 \rightarrow \boxed{x = 22}$

$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{t}}}{\frac{3\sqrt{t}}{2}} = \frac{2}{6t} = \frac{1}{3t}$ $\quad y = t^{1/2}$

$\frac{dx}{dt} = \frac{3}{2}t^{1/2} = \frac{3\sqrt{t}}{2}$

$\frac{dy}{dt} = \frac{1}{2}t^{-1/2} = \frac{1}{2\sqrt{t}}$

6. A parametric curve is defined by the equations $x = \ln(t)$ and $y = \arctan(t)$, use the equation of the tangent line to approximate the value at t=1. $\quad$ at $t = 1$

$y - \frac{\pi}{4} = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + \frac{\pi}{4}$

$y = \frac{1}{2}\ln(4) + \frac{\pi}{4}$

$\boxed{y = \ln(2) + \frac{\pi}{4}}$

$\frac{dy}{dx} = \frac{\frac{1}{1+t^2}}{\frac{1}{t}} = \frac{t}{1+t^2}$

$\frac{1}{1+1^2} = \frac{1}{2}$