Analyze your name on this line: Key

AP Calculus BC: 12.3 Function Analysis of Parametric Equations

1. Parametric equations are defined by, $x = t^2 - t + 1$ and $y = 3t^3$, for the interval $[0,5]$,
   a) Find the horizontal and vertical tangent lines. Show answers in terms of t and points (x,y).

$\frac{dy}{dx} = \frac{9t^2 = 0}{2t-1=0}$ $\quad 9t^2 = 0$, $t = 0$ HTL, $(1,0)$ $\quad 2t-1=0$, $t = \frac{1}{2}$ VTL, $(\frac{3}{4}, \frac{3}{8})$ $\quad (\frac{1}{2})^2 - \frac{1}{2} + 1$, $\frac{1}{4} - \frac{2}{4} + \frac{4}{4} = \frac{3}{4}$ $\quad 3(\frac{1}{2})^3$, $\frac{3}{8}$

   b) When is the parametric equations increasing, in terms of t?

$[\frac{1}{2}, 5]$

   c) Find the interval when the function is moving to the left of the horizontal axis, in terms of t.

$\frac{dx}{dt} = 2t - 1$ $\quad 2t-1=0$, $t = \frac{1}{2}$

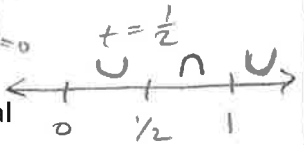
$[0, 1/2]$

   d) When is the parametric equations concave up, in terms of t?

$(\frac{1}{2}, 1)$

$\frac{d^2y}{dx^2} = \frac{\frac{(2t-1)18t - 9t^2(2)}{(2t-1)^2}}{2t-1} = \frac{(2t-1)18t - 9t^2(2)}{(2t-1)^3} = \frac{36t^2 - 18t - 18t^2}{(2t-1)^3} = \frac{18t(t-1)}{(2t-1)^3} = 0$

$18t^2 - 18t$, $18t(t-1)$, $t=0$ $t=1$



2. The parametric equations, $x = \sin(2t)$ and $y = 2\cos(t)$, are defined for the interval $0 \le t \le \pi$.
   a) Find all vertical and horizontal tangency to the parametric equation. Show answers in terms of t ~~and points (x,y)~~.

$\frac{dy}{dx} = \frac{-2\sin(t) = 0}{2\cos(2t) = 0}$ $\quad -2\sin(t) = 0$, $\sin(t) = 0$, $t = 0, \pi$ HTL $\quad 2\cos(2t) = 0$, $\cos(2t) = 0$, $2t = \pi/2$ $\quad 2t = \frac{3\pi}{2}$, $t = \pi/4$ $\quad t = \frac{3\pi}{4}$, VTL

   b) Find the equation of the tangent line for the function at $t = \frac{\pi}{6}$

$y - \sqrt{3} = -\sqrt{3}(x - \frac{\sqrt{3}}{2})$

$y = -\sqrt{3}(x - \frac{\sqrt{3}}{2}) + \sqrt{3}$

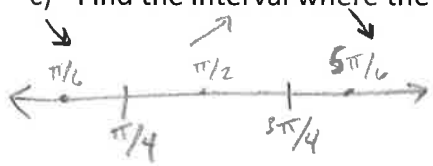
$x = \sin(\frac{2\pi}{6}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$y = 2\cos(\pi/6) = \frac{2}{1}\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

$\frac{dy}{dx} = \frac{-2\sin(\pi/6)}{2\cos(2\pi/6)} = \frac{-\sin(\pi/6)}{\cos(\pi/3)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$

   c) Find the interval where the parametric curve is decreasing, in terms of t.

S | A
T | C



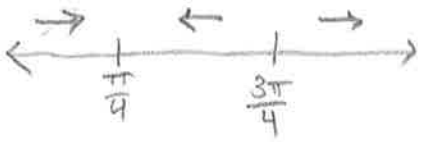
$\frac{-2\sin(t)}{2\cos(2t)}$

$[0, \pi/4]$ $[\frac{3\pi}{4}, \pi]$

   d) Find the interval, on the horizontal axis, where the graph is moving to the left, in terms of t.

$\frac{dx}{dt} = 2\cos(2t) = 0$

$t = \frac{\pi}{4}$ & $\frac{3\pi}{4}$



$[\pi/4, \frac{3\pi}{4}]$