Log your name in the line: Key

AP Calculus BC: 11.4 Logistic Growth

1. (Calculator) Populations have logistical growth due to the inability to sustain an infinite amount of species. The initial mammoth population was 150. Five decades later their population grew to 310. If the Earth can only support around 700 mammoths,
   a. Create a differential equation and find its particular solution for the growth of the mammoth population, if time is measured in decades.

$\frac{dP}{dt} = kP(700-P)$ $\quad P(0)=150,\ P(5)=310 \Rightarrow C = 11/3,\ k = 3.05\times10^{-4}$

$P = \frac{700}{1+Ce^{-700kt}}$

   b. When does the Mammoth population grow the fastest? Justify.

$P''(t)=0$

$\boxed{t = 6.086 \text{ decades}}$

   c. When was the Mammoth's population growing fastest after 4 or 7 decades? Justify.

$P'(4) = 35.57 \frac{\text{mam}}{\text{dec}}$

$P'(7) = 37.009 \frac{\text{mam}}{\text{dec}}$

Growing faster at $t=7$ decades because the $P'(7)$ is greater than $P'(4)$

   d. When does the Mammoth population reach 550?

$P(t) = 550$

$\boxed{t = 12.171 \text{ decades}}$

   e. When was the Mammoth's population growing fastest, when there were 250 or 550 Mammoths alive? Justify.

$\frac{dP}{dt} = 3.05\times10^{-4} \cdot 250(700-250) = 34.313 \frac{\text{mamoths}}{\text{decade}}$

$\frac{dP}{dt} = 3.05\times10^{-4} \cdot 550(700-550) = 25.163 \frac{\text{mammoths}}{\text{decade}}$

growing fastest at 250 mammoths because the rate of growth is larger than 550 mammoths

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2. (Calculator) The Joe Rogan Podcast is one of the most popular podcasts around. But it wasn't always like that. It started small with only a few listeners and as they spread the news of the podcast it gained traction and had logistic growth. If the podcast started with only around 1 million people and now, after 5 years, has around 16 million. If there are 1,300,000,000 podcast listeners,
   a. Create a differential equation and find its particular solution for the Joe Rogan Podcast growth, if time is measured in years.

$\frac{dP}{dt} = kP(1.3\times10^9 - P)$ $\quad P(0) = 1\times10^6 \quad \Rightarrow \quad C = 1299$

$P = \frac{1.3\times10^9}{1+Ce^{-1.3\times10^9 kt}}$ $\quad P(5) = 16\times10^6 \quad k = 4.283\times10^{-10}$

   b. How many people will be listening to the Joe Rogan Podcast in 5 more years?

$P(10) = 218{,}112{,}768$ people

   c. When did the podcast have its fastest growth?

$P''(t) = 0$

$\boxed{t = 12.876 \text{ years}}$

   d. Find when Joe Rogan's podcast had 150 million listeners?

$P(t) = 150\times10^6$

$\boxed{t = 9.218 \text{ years}}$

   e. When was the podcast growing faster when there were 200 million or 500 million listeners? Justify.

$\frac{dP}{dt} = k(200\times10^6)(1.3\times10^9 - 200\times10^6) = 94{,}226{,}000 \frac{\text{listeners}}{\text{year}}$

$\frac{dP}{dt} = k(500\times10^6)(1.3\times10^9 - 500\times10^6) = 171{,}320{,}000 \frac{\text{listeners}}{\text{year}}$

$\boxed{\text{growing faster at 500 million listeners}}$