Approximate your name: Key

AP Calculus BC: 11.3 Euler's Method of Approximation

1. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - 2y$ with an initial condition $f(7) = 2$. What is an approximation for $f(8)$ by using Euler's method with two steps of equal length? *(Ans: ~~0~~)*

   $f(7.5) \approx f(7) + f'(7)(.5) = 2 + 3(.5) = 3.5$

   $f(8) \approx f(7.5) + f'(7.5)(.5) = 3.5 + (14.5)(.5) = \boxed{10.75}$

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = y - x^2$ with an initial condition $f(0) = 2$. What is an approximation for $f(1)$ by using Euler's method with ~~three~~ two steps of equal length? *(Ans: 4.375)*

   $f(0.5) \approx f(0) + f'(0)(.5) = 2 + 2(.5) = 3$

   $f(1) \approx f(.5) + f'(.5)(.5) = 3 + 2.75(.5) = \boxed{4.375}$

3. Let $y = f(x)$ be a solution to the differential equation $f'(x)$ with an initial condition $f(1.5) = 4$. What is an approximation for $f(3)$ by using Euler's method with three steps of equal length? *(Ans: 11.5)*

x	1	1.5	2	2.5	3
f'(x)	3	4	6	5	2

   $f(2) \approx f(1.5) + f'(1.5)(.5) = 4 + (4)(.5) = 6$

   $f(2.5) \approx f(2) + f'(2)(.5) = 6 + 6(.5) = 9$

   $f(3) \approx f(2.5) + f'(2.5)(.5) = 9 + 5(.5) = \boxed{11.5}$

4. Calculator Problem. The differential equation $\frac{dL}{dt} = t^2(L - 4)$ where L represents the amount of water that is leaking out of a tank of water and t represents the time, in minutes. Initially there were 7 liters that had left the tank, use Euler's method to approximate how many liters left the tank after 1 minute with two step sizes of equal length.

   $f(.5) \approx f(0) + f'(0)(.5) = 7 + 0(.5) = 7$

   $f(1) \approx f(.5) + f'(.5)(.5) = 7 + .75 = \boxed{7.75}$

b. Find the particular solution of the differential equation to calculate the exact amount of liters that had left the tank.

$\int \frac{1}{L-4} = \int t^2 dt$

$\ln|L-4| = \frac{t^3}{3} + C$

$\ln|3| = C$

$C = \ln(3)$

$\ln|L-4| = \frac{t^3}{3} + \ln(3)$

$L - 4 = e^{t^3/3 + \ln(3)}$

$L - 4 = 3e^{t^3/3}$

$\boxed{L = 3e^{t^3/3} + 4}$

$L(1) = 3e^{1/3} + 4$

$\boxed{L(1) = 8.187 \text{ liters}}$