Revolve your pencil around the paper to create your name: key

AP Calculus BC: 11.2 Surface of Revolution

1. Set up an integral to find the surface of revolution for the function $f(x) = \arcsin(4x)$ on the interval $[0, \frac{\pi}{6}]$ around the x-axis.

$$2\pi \int_0^{\pi/6} \arcsin(4x)\sqrt{1+\left(\frac{4}{\sqrt{1-(4x)^2}}\right)^2}\,dx$$

2. Set up an integral to find the surface of revolution for the function $f(x) = \ln(3x)$ from x=1 to x=4 around the x-axis.

$$2\pi \int_1^4 \ln(3x)\sqrt{1+\left(\frac{1}{x}\right)^2}\,dx$$

b. Set up an integral to find the surface of revolution for the function $f(x) = \ln(3x)$ from x=1 to x=4 around the y-axis. (notice this question is similar to the one above)

$y = \ln(3x)$

$e^y = 3x$

$x = \frac{e^y}{3}$

$$2\pi \int_{e/3}^{e^4/3} \frac{e^y}{3}\sqrt{1+\left(\frac{e^y}{3}\right)^2}\,dy$$

3. Find the surface of revolution of the function $f(x) = \sqrt{x}$ on the interval [0,1] around the x-axis.

$f' = \frac{1}{2\sqrt{x}}$

$$2\pi\int_0^1 \sqrt{x}\sqrt{1+\left(\frac{1}{2\sqrt{x}}\right)^2}\,dx = 2\pi\int_0^1 \sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = 2\pi\int_0^1 \sqrt{x\left(1+\frac{1}{4x}\right)} = 2\pi\int_0^1 \sqrt{x+\frac{1}{4}}\,dx$$

$u = x + \frac{1}{4}$

$du = dx$

$$2\pi\int_{1/4}^{5/4} (u)^{1/2}\,du = 2\pi\left[\frac{2u^{3/2}}{3}\right]_{1/4}^{5/4} = 2\pi\left[\frac{2\left(\frac{5}{4}\right)^{3/2}}{3} - \frac{2\left(\frac{1}{4}\right)^{3/2}}{3}\right] = 2\pi\left[\frac{2\sqrt{5^3}}{24} - \frac{2}{24}\right] = \frac{2\pi(2\sqrt{5^3}-2)}{24}$$

$$= \frac{4\pi(\sqrt{5^3}-1)}{24} = \boxed{\frac{\pi(\sqrt{5^3}-1)}{6}}$$

4. Using a calculator find the surface of revolution for the function $f(x) = e^{2x}$ on the interval of [0,4].

$$2\pi\int_0^4 f(x)\sqrt{1+(f'(x))^2}\,dx = 169.932$$

$(4-x^2)^{1/2}$

$\frac{-2x}{2(4-x^2)^{1/2}}$

$y = \sqrt{4-x^2}$

$y' = \frac{-2x}{2\sqrt{4-x^2}}$

5. A sphere with radius 2 has a surface area of 16π units squared. The formula for a semi circle of radius 2 is $x^2 + y^2 = 4$. Set-up an equation and prove the surface area is 16π using solids of revolution ~~using a calculator~~.

$$2\pi\int_{-2}^2 \sqrt{4-x^2}\sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\,dx = 4\pi\int_0^2 \sqrt{4-x^2}\sqrt{1+\frac{x^2}{4-x^2}}\,dx = 4\pi\int_0^2 \sqrt{(4-x^2)\left(1+\frac{x^2}{4-x^2}\right)}\,dx$$

$$= 4\pi\int_0^2 (4-\cancel{x^2}+\cancel{x^2})\,dx = 4\pi\int_0^2 4\,dx = 4\pi\left[4x\right]_0^2 = 4\pi[8-0] = \boxed{16\pi}$$