Put your name on the length of this curve: key

AP Calculus BC: 11.1 Length of a Curve

1. Set up an integral to find the length of a curve for the function $f(x) = \cos(2x)$ on the interval $[0, \frac{\pi}{6}]$

$f' = -\sin(2x)\cdot 2 = -2\sin(2x)$

$$\int_0^{\pi/6} \sqrt{1+(-2\sin(2x))^2}\,dx$$

2. Set up an integral to find the length of a curve for the function $f(x) = \ln(5x)$ from x=1 to x=4.

$f' = \frac{5}{5x} = \frac{1}{x}$

$$\int_1^4 \sqrt{1+\left(\frac{1}{x}\right)^2}\,dx$$

3. Find the exact length of the curve of the function $f(x) = 5x + 3$ from x=0 to x=10.

$f' = 5$

$$\int_0^{10} \sqrt{1+(5)^2}\,dx = \int_0^{10} \sqrt{26}\,dx = \left[\sqrt{26}x\right]_0^{10} = \boxed{10\sqrt{26}}$$

4. Find the exact length of the curve of the function $f(x) = 2x^{\frac{3}{2}} + 7$ for the interval [0,3].

$f' = 3x^{1/2}$

$$\int_0^3 \sqrt{1+(3\sqrt{x})^2}\,dx = \int_0^3 \sqrt{1+9x}\,dx$$

$u = 1+9x$

$du = 9\,dx$

$dx = \frac{du}{9}$

$$= \int_1^{28} \frac{u^{1/2}}{9}\,du = \left[\frac{(2)u^{3/2}}{9(3)}\right]_1^{28} = \frac{2\sqrt{28^3}}{27} - \frac{2\sqrt{1^3}}{27}$$

$$\boxed{\frac{2\sqrt{28^3}-2}{27}}$$

Use a calculator beyond this point.

5. Find the exact length of a curve for the function $f(x) = e^{2x}$ on the interval of [0,4].

$f' = 2e^{2x}$

$$\int_0^4 \sqrt{1+(2e^{2x})^2}\,dx = \boxed{2980.081}$$

6. Find the length of the curve $y = x^2 - 5x + 1$ for x=1 to x=5.

$$\int_1^5 \sqrt{1+(2x-5)^2}\,dx = \boxed{9.778}$$