Who has infinite bounds of potential → That guy: Key

AP Calculus BC: 10.8 Improper Integrals with Infinite Bounds

Evaluate each integral below

1. $\int_3^\infty \frac{2}{x}\,dx$ $\quad \lim_{x\to\infty} \frac{2}{x} = 0$ converges

$\lim_{b\to\infty}\int_3^b \frac{2}{x}\,dx = \lim_{b\to\infty}\left[2\ln|x|\right]_3^b$

$\lim_{b\to\infty} 2\ln|b| - 2\ln|3|$

$\infty - 2\ln|3| = \boxed{\infty}$

$\boxed{\text{divergent}}$

2. $\int_2^\infty \frac{4}{\sqrt{x^3}}\,dx$ $\quad \lim_{x\to\infty} \frac{4}{\sqrt{x^3}} = 0$ converges

$\lim_{b\to\infty}\int_2^b 4x^{-3/2}\,dx = \lim_{b\to\infty}\left[\frac{(2)4x^{-1/2}}{-1}\right]_2^b$

$\lim_{b\to\infty}\left[\frac{8}{\sqrt{x}}\right]_2^b = \lim_{b\to\infty}\cancel{\frac{8}{\sqrt{b}}}^{\,0} - \frac{-8}{\sqrt{2}} = \frac{+8}{\sqrt{2}} = \boxed{4\sqrt{2}}$

3. $\int_1^\infty \frac{2}{\sqrt{x}}\,dx$ $\quad \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0$ converges

$\lim_{b\to\infty}\int_1^b 2x^{-1/2}\,dx = \lim_{b\to\infty}\left[\frac{(2)2x^{1/2}}{1}\right]_1^b$

$\lim_{b\to\infty} 4\sqrt{b} - 4\sqrt{1} = 4\sqrt{\infty} - 4 = \boxed{\infty}$

$\boxed{\text{divergent}}$

4. $\int_1^\infty \frac{x^2+4}{3x}\,dx$

$\lim_{x\to\infty} \frac{x^2+4}{3x} = \boxed{\infty}$

$\boxed{\text{divergent}}$

5. $\int_9^\infty \frac{6x^2}{x^3-2}\,dx$ $\quad \lim_{x\to\infty} \frac{6x^2}{x^3-2} = 0$ convergent

$u = x^3-2$
$du = 3x^2\,dx$
$dx = \frac{du}{3x^2}$

$\int_9^\infty \frac{\cancel{6x^2}^{\,2}}{u}\,\frac{du}{\cancel{3x^2}}$

$\int_9^\infty \frac{2}{u}\,du = \left[2\ln|u|\right]_9^\infty = \left[2\ln|x^3-2|\right]_9^\infty$

$\lim_{b\to\infty}\left[2\ln|x^3-2|\right]_9^\infty = \lim_{b\to\infty} 2\ln|b^3-2| - 2\ln|9^3-2| = \boxed{\infty}$

$\boxed{\text{divergent}}$

6. $\int_{-\infty}^{-3} \frac{6}{(2x+4)^2}\,dx$ $\quad \lim_{x\to-\infty} \frac{6}{(2x+4)^2} = 0$ convergent

$u = 2x+4$
$du = 2\,dx$
$dx = \frac{du}{2}$

$\int_{-\infty}^{-3} \frac{\cancel{6}\,3}{u^2}\,\frac{du}{\cancel{2}} = \int_{-\infty}^{-3} \frac{3}{u^2}\,du = \int_{-\infty}^{-3} 3u^{-2}\,du$

$\left[\frac{3u^{-1}}{-1}\right]_{-\infty}^{-3} = \left[\frac{-3}{u}\right]_{-\infty}^{-3}$

$\lim_{a\to-\infty}\left[\frac{-3}{2x+4}\right]_a^{-3} = \frac{-3}{2(-3)+4} - \lim_{a\to-\infty}\cancel{\frac{-3}{2x+4}}^{\,0}$

$\boxed{\frac{3}{2}}$

7. $\int_{-\infty}^{2} e^{3x}\,dx$ $\quad \lim_{x\to-\infty} e^{3x} = 0$ convergent

$\left[\frac{e^{3x}}{3}\right]_{-\infty}^2 = \lim_{a\to-\infty}\left[\frac{e^{3x}}{3}\right]_a^2$

$\frac{e^6}{3} - \lim_{a\to-\infty}\cancel{\frac{e^{3a}}{3}}^{\,0} = \boxed{\frac{e^6}{3}}$

8. $\int_0^\infty 3x^2e^{-x}\,dx$ $\quad \lim_{x\to\infty} 3x^2e^{-x} = 0$ convergent

+	$3x^2$	e^{-x}
−	$6x$	$-e^{-x}$
+	6	e^{-x}
−	0	$-e^{-x}$

$\left[-3x^2e^{-x} - 6xe^{-x} - 6e^{-x}\right]_0^\infty$

$\lim_{x\to\infty} \cancel{-3x^2e^{-x} - 6xe^{-x} - 6e^{-x}}^{\,0} - (\cancel{-3(0)^2e^0} - \cancel{6(0)e^0} - 6e^0) = \boxed{6}$