Put your name up here yo! Key

AP Calculus BC: 10.6 Integration by Parts Repeatedly

1. $\int \cos(x)\, e^{2x}\, dx$

$u = \cos(x) \qquad v = \frac{e^{2x}}{2}$

$du = -\sin(x)\,dx \qquad dv = e^{2x}\,dx$

$\frac{e^{2x}\cos(x)}{2} + \int \frac{e^{2x}\sin(x)}{2}\,dx$

$u = \sin(x) \qquad v = \frac{e^{2x}}{4}$

$du = \cos(x)\,dx \qquad dv = \frac{e^{2x}}{2}\,dx$

$\int \cos(x) e^{2x}\,dx = \frac{e^{2x}\cos(x)}{2} + \left(\frac{e^{2x}\sin(x)}{4} - \int \frac{e^{2x}\cos(x)}{4}\,dx\right)$

$+\frac{1}{4}\int \cos(x) e^{2x}\,dx \qquad + \int \frac{e^{2x}\cos(x)}{4}\,dx$

$\frac{5}{4}\int \cos(2x) e^{2x} = \frac{e^{x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{4}$

$\int \cos(2x) e^{2x} = \boxed{\frac{4}{5}\left(\frac{e^{x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{4}\right)}$

2. Find the integral of the function $f(x) = e^{4x}\sin(2x)$

$u = \sin(2x) \qquad v = \frac{e^{4x}}{4}$

$du = 2\cos(2x)\,dx \qquad dv = e^{4x}\,dx$

$\frac{e^{4x}\sin(2x)}{4} - \int \frac{e^{4x}\cos(2x)}{2}\,dx$

$u = \cos(2x) \qquad v = \frac{e^{4x}}{8}$

$du = -2\sin(2x) \qquad dv = \frac{e^{4x}}{2}\,dx$

$\int e^{4x}\sin(2x)\,dx = \frac{e^{4x}\sin(2x)}{4} - \left(\frac{e^{4x}\cos(2x)}{8} + \int \frac{e^{4x}\sin(2x)}{4}\,dx\right)$

$+\frac{1}{4}\int e^{4x}\sin(2x)\,dx \qquad +\frac{1}{4}\int e^{4x}\sin(2x)\,dx$

$\frac{5}{4}\int e^{4x}\sin(2x)\,dx = \frac{e^{4x}\sin(2x)}{4} - \frac{e^{4x}\cos(2x)}{8}$

$\int e^{4x}\sin(2x)\,dx = \boxed{\frac{4}{5}\left(\frac{e^{4x}\sin(2x)}{4} - \frac{e^{4x}\cos(2x)}{8}\right)}$

3. Calculate the area of the function $g(x) = e^{2x}\cos(x)$ on the interval $[0,\pi]$

$u = \cos(x) \qquad v = \frac{e^{2x}}{2}$

$du = -\sin(x)\,dx \qquad dv = e^{2x}\,dx$

$\frac{e^{2x}\cos(x)}{2} + \int \frac{e^{2x}\sin(x)}{2}\,dx$

$u = \sin(x) \qquad v = e^{2x}/4$

$du = \cos(x)\,dx \qquad dv = \frac{e^{2x}}{2}\,dx$

$\downarrow$

$\int e^{2x}\cos(x)\,dx = \frac{e^{2x}\cos(x)}{2} + \left(\frac{e^{2x}\sin(x)}{4} - \int \frac{e^{2x}\cos(x)}{4}\,dx\right)$

$\int e^{2x}\cos(x)\,dx = \frac{e^{2x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{4} - \frac{1}{4}\int e^{2x}\cos(x)\,dx$

$+\frac{1}{4}\int e^{2x}\cos(x)\,dx \qquad +\frac{1}{4}\int e^{2x}\cos(x)\,dx$

$\frac{5}{4}\int e^{2x}\cos(x)\,dx = \frac{e^{2x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{4}$

$\int e^{2x}\cos(x)\,dx = \frac{4}{5}\left(\frac{e^{2x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{5}\right)$

$\left[\frac{4}{5}\left(\frac{e^{2x}\cos(x)}{2} + \frac{e^{2x}\sin(x)}{4}\right)\right]_0^{\pi}$

$\left[\frac{4}{5}\left(\frac{e^{2\pi}\overset{-1}{\cos(\pi)}}{2} + \cancel{\frac{e^{2\pi}\overset{0}{\sin(\pi)}}{4}}\right)\right] - \left[\frac{4}{5}\left(\frac{\overset{1}{e^{0}}\overset{1}{\cos(0)}}{2} + \cancel{\frac{e^{0}\overset{0}{\sin(0)}}{4}}\right)\right]$

$\frac{-4e^{2\pi}}{10} - \frac{4}{10} = \boxed{-\frac{2e^{2\pi}}{5} - \frac{2}{5}}$