Put your name in parts then u sub it here: Key

AP Calculus BC: 10.5 Integration by Parts w/ U-sub

Find the integral of each of the following functions. I mixed in a regular u-sub in there again but it can be done with integration by parts too, so let's see if you catch it. ENJOY!

1. $\int 6x\,e^{3x}\,dx$

$u = 6x \qquad v = \frac{e^{3x}}{3}$

$du = 6\,dx \qquad dv = e^{3x}\,dx$

$\frac{6xe^{3x}}{3} - \int \frac{6e^{3x}}{3}\,dx$

$2xe^{3x} - \int 2e^{3x}\,dx$

$u = 3x \quad du = 3\,dx \quad dx = \frac{du}{3}$

$\boxed{2xe^{3x} - \frac{2e^{3x}}{3} + C}$

2. $\int \frac{2x}{e^{6x}}\,dx$

$u = 2x \qquad v = \frac{e^{-6x}}{-6}$

$du = 2\,dx \qquad dv = e^{-6x}\,dx$

$\frac{2xe^{-6x}}{-6} - \int \frac{2e^{-6x}}{-6}\,dx$

$\frac{-x}{6e^{6x}} - \int \frac{e^{-6x}}{-3}\,dx$

$u = -6x \quad du = -6\,dx \quad dx = \frac{du}{-6}$

$\frac{-x}{6e^{6x}} - \frac{e^{-6x}}{18} + C$

$\boxed{\frac{-x}{6e^{6x}} - \frac{1}{18e^{6x}} + C}$

5. $\int x\sec^2(x)\,dx$

$u = x \qquad v = \tan(x)$

$du = dx \qquad dv = \sec^2(x)\,dx$

$x\tan(x) - \int \tan(x)\,dx$

$x\tan(x) - \int \frac{\sin(x)}{\cos(x)}\,dx$

$u = \cos(x) \quad du = -\sin(x)\,dx \quad dx = \frac{du}{-\sin(x)}$

$x\tan(x) - \int \frac{\sin(x)}{u}\,\frac{du}{-\sin(x)}$

$x\tan(x) + \int \frac{1}{u}\,du = \boxed{x\tan(x) + \ln|\cos(x)| + C}$

6. $\int \arccos(2x)\,dx$

$u = \arccos(2x) \qquad v = x$

$du = \frac{-2}{\sqrt{1-4x^2}}\,dx \qquad dv = dx$

$x\arccos(2x) - \int \frac{-2x}{\sqrt{1-4x^2}}\,dx$

$u = 1-4x^2 \quad du = -8x\,dx \quad dx = \frac{du}{-8x}$

$x\arccos(2x) - \int \frac{-2x}{u^{1/2}}\,\frac{du}{-8x}$

$x\arccos(2x) - \int \frac{u^{-1/2}}{4}\,du$

$x\arccos(2x) - \frac{2u^{1/2}}{4} + C$

$\boxed{x\arccos(2x) - \frac{\sqrt{1-4x^2}}{2} + C}$

7. Calculate $\int_0^1 4x\,e^{2x}\,dx$

$u = 4x \qquad v = \frac{e^{2x}}{2}$

$du = 4\,dx \qquad dv = e^{2x}\,dx$

$u = 2x \quad du = 2\,dx \quad dx = \frac{du}{2}$

$\left[\frac{4xe^{2x}}{2}\right]_0^1 - \int_0^1 \frac{4e^{2x}}{2}\,dx = \left[2xe^{2x}\right]_0^1 - \int 2e^{2x}\,dx = \left[2xe^{2x} - e^{2x}\right]_0^1$

$\left[(2(1)e^{2(1)} - e^{2(1)}) - (2(0)e^{2(0)} - e^{2(0)})\right]$

$2e^2 - e^2 - 0 + e^0$

$\boxed{e^2 + 1}$