U substitute your name for the line: Key

AP Calculus AB: Advanced U-Substitution

1. $\int \frac{3x\sqrt{x^2+4}}{2}dx$

$u=x^2+4$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int \frac{3x(u)^{1/2}}{2}\frac{du}{2x}$

$\int \frac{3u^{1/2}}{4}du = \frac{3u^{3/2}}{\frac{4}{\frac{3}{2}}} = \frac{6u^{3/2}}{\cancel{12}} = u^{3/2}$

$\boxed{\frac{(x^2+4)^{3/2}}{2}+C}$

2. $\int \frac{12(\sqrt{x}+4)^5}{\sqrt{x}}dx$

$u=x^{1/2}+4$

$du=\frac{1}{2}x^{-1/2}dx$

$du=\frac{dx}{2\sqrt{x}}$

$dx=2\sqrt{x}\,du$

$\int \frac{12u^5}{\cancel{\sqrt{x}}}2\cancel{\sqrt{x}}du$

$\int 24u^5du$

$\frac{24u^6}{6} = 4u^6+C = \boxed{4(\sqrt{x}+4)^6+C}$

3. $\int \frac{x}{(x^2+5)^3}dx$

$u=x^2+5$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int \frac{\cancel{x}}{u^3}\frac{du}{2\cancel{x}}$

$\int \frac{u^{-3}}{2}du = \frac{u^{-2}}{\frac{2}{\frac{-2}{1}}} = \frac{u^{-2}}{-4} = \frac{1}{-4u^2}$

$\boxed{\frac{1}{-4(x^2+5)^2}+C}$

4. $\int \frac{6x}{\sqrt[3]{x^2+3}}dx$

$u=x^2+3$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int \frac{\overset{3}{\cancel{6x}}}{u^{1/3}}\frac{du}{\cancel{2x}}$

$\int \frac{3}{u^{1/3}}du = \int 3u^{-1/3}du = \frac{3u^{2/3}}{\frac{2}{3}} = \boxed{\frac{9(x^2+3)^{2/3}}{2}+C}$

5. The derivative of a function is given by $f'(x) = 6x(x^2+1)^2$. If the original function passes through the point (1,10)

(a) Find the particular solution of f(x).

$u=x^2+1$

$du=2x\,dx$

$dx=\frac{du}{2x}$

$\int \overset{3}{\cancel{6x}}(u)^2\frac{du}{\cancel{2x}}$

$\int 3u^2du$

$\frac{3u^3}{\cancel{3}}+C = (x^2+1)^3+C$

$f(x)=(x^2+1)^3+C$

$10=(1^2+1)^3+C$

$10=8+C$

$C=2$

$\boxed{f(x)=(x^2+1)^3+2}$

(b) Find f(0).

$f(0)=(0^2+1)^3+2=\boxed{3}$

6. $\int x(x-4)^2dx$

$u=x-4$

$du=dx$

$x=u+4$

$\int (u+4)u^2du$

$\int u^3+4u^2du$

$\frac{u^4}{4}+\frac{4u^3}{3}+C$

$\boxed{\frac{(x-4)^4}{4}+\frac{4(x-4)^3}{3}+C}$

7. $\int \frac{x}{(x+2)^3}dx$

$u=x+2$

$du=dx$

$x=u-2$

$\int \frac{u-2}{u^3}du$

$\int u^{-2}-2u^{-3}du$

$\frac{u^{-1}}{-1}-\frac{\cancel{2}u^{-2}}{-\cancel{2}}+C$

$\boxed{-\frac{1}{x+2}+\frac{1}{(x+2)^2}+C}$